S700. Proposed by An Zhenping, Xianyang Normal University, China

Let a,b,c,x,y,z be positive real numbers. Prove that

$$\frac{x}{y+z}(b^3+c^3)+\frac{y}{z+x}(c^3+a^3)+\frac{z}{x+y}(a^3+b^3)\geq 3abc$$

Solution by Arkady Alt, San Jose, California, USA.

Since for any $a,b,c,x,y,z>0$ holds inequality

$$\frac{x(b+c)}{y+z}+\frac{y(c+a)}{z+x}+\frac{z(a+b)}{x+y}\geq\sqrt{3(ab+bc+ca)}$$ (Inequality (**F3**) in [1])

then by replacing (a,b,c) with (a^3,b^3,c^3) we obtain

$$\frac{x}{y+z}(b^3+c^3)+\frac{y}{z+x}(c^3+a^3)+\frac{z}{x+y}(a^3+b^3)\geq\sqrt{3(a^3b^3+b^3c^3+c^3a^3)}$$

and also $\sqrt{3(a^3b^3+b^3c^3+c^3a^3)}\geq 3abc$ because by AM-GM inequality

$a^3b^3+b^3c^3+c^3a^3\geq 3(a^3b^3\cdot b^3c^3\cdot c^3a^3)^{1/3}=3a^2b^2c^2.$

[1] **Variations on theme of one algebraic inequality_Arkady Alt** ,
Mathematical Reflections n.2, 2009, p.6.
See by link:
https://www.academia.edu/32055493/Variations_on_an_algebraic_inequality_
Mathematical_Reflections_n_2_2009