Problems

Ted Eisenberg, Section Editor

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This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <http://www.ssma.org/publications>.

Solutions to the problems stated in this issue should be posted before
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- **5409:** *Proposed by Kenneth Korbin, New York, NY*

  Given isosceles trapezoid $ABCD$ with $\overline{AB} < \overline{CD}$, and with diagonal $\overline{AC} = \overline{AB} + \overline{CD}$. Find the perimeter of the trapezoid if $\triangle ABC$ has inradius 12 and if $\triangle ACD$ has inradius 35.

- **5410:** *Proposed by Arkady Alt, San Jose, CA*

  For the given integers $a_1, a_2, a_3 \geq 2$ find the largest value of the integer semiperimeter of a triangle with integer side lengths t_1, t_2, t_3 satisfying the inequalities $t_i \leq a_i,\ i = 1, 2, 3$.

- **5411:** *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" General School, Buzău, Romania*

  Let $(a_n)_{n\geq 1}, (b_n)_{n\geq 1}$ be real valued positive sequences with $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} b_n = a \in R_+^*$
  If $\lim\limits_{n\to\infty} (n(a_n - a)) = b \in R$ and $\lim\limits_{n\to\infty} (n(b_n - a)) = c \in R$ compute

  $$\lim_{n\to\infty} \left(a_{n+1} \sqrt[n+1]{(n+1)!} - b_n \sqrt[n]{n!} \right).$$

  Note: R_+^* means the positive real numbers without zero.

- **5412:** *Proposed by Michal Kremzer, Gliwice, Silesia, Poland*

  Given positive integer M. Find a continuous, non-constant function $f : R \to R$ such that $f(f(x)) = f([x])$, for all real x, and for which the maximum value of $f(x)$ is M.
  Note: $[x]$ is the greatest integer function.

- **5413:** *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*

  Compute

  $$\lim_{n\to\infty} \frac{1}{n} \sum_{1\leq i\leq j\leq n} \frac{1}{\sqrt{(n^2 + (i+j)n + ij)}}.$$