

Problems

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This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <<http://www.ssma.org/publications>>.

*Solutions to the problems stated in this issue should be posted before
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- **5403:** *Proposed by Kenneth Korbin, New York, NY*

Let $\phi = \frac{1 + \sqrt{5}}{2}$. Solve the equation $\sqrt[3]{x + \phi} = \sqrt[3]{\phi} + \sqrt[3]{x - \phi}$ with $x > \phi$.

- **5404:** *Proposed Arkady Alt, San Jose, CA*

For any given positive integer $n \geq 3$, find the smallest value of the product of $x_1 x_2 \dots x_n$, where $x_1, x_2, x_3, \dots, x_n > 0$ and $\frac{1}{1 + x_1} + \frac{1}{1 + x_2} + \dots + \frac{1}{1 + x_n} = 1$.

- **5405:** *Proposed by D. M. Bătinetu-Giurgiu, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania*

If $a, b \in \mathfrak{R}$ such that $a + b = 1$, $e_n = \left(1 + \frac{1}{n}\right)^n$ and $c_n = -\ln n + \sum_{k=1}^n \frac{1}{k}$, then compute

$$\lim_{n \rightarrow \infty} \left((n+1)^a \sqrt[n+1]{((n+1)!c_n)^b} - n^a \sqrt[n]{(n!e_n)^b} \right).$$

- **5406:** *Proposed by Cornel Ioan Vălean, Timis, Romania*

Calculate:

$$\sum_{n=1}^{\infty} \frac{H_n}{n} \left(\zeta(3) - 1 - \frac{1}{2^3} - \dots - \frac{1}{n^3} \right),$$

where $H_n = \sum_{k=1}^n \frac{1}{k}$ denotes the harmonic number.

- **5407:** *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*