Problems

Ted Eisenberg, Section Editor

This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at http://www.ssma.org/publications>.

Solutions to the problems stated in this issue should be posted before April 15, 2015

• 5337: Proposed by Kenneth Korbin, New York, NY

Given convex quadrilateral ABCD with sides,

$$\overline{AB} = 1 + 3\sqrt{2}$$

$$\overline{BC} = 6 + 4\sqrt{2} \text{ and}$$

$$\overline{CD} = -14 + 12\sqrt{2}.$$

Find side \overline{AD} so that the area of the quadrilateral is maximum.

• 5338: Proposed by Arkady Alt, San Jose, CA Determine the maximum value of

$$F(x, y, z) = \min \left\{ \frac{|y - z|}{|x|}, \frac{|z - x|}{|y|}, \frac{|x - y|}{|z|} \right\},$$

where x, y, z are arbitrary nonzero real numbers.

• 5339: Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu "George Emil Palade" School, Buzău, Romania

Calculate:
$$\int_0^{\pi/2} \frac{3\sin x + 4\cos x}{3\cos x + 4\sin x} dx.$$

• 5340: Proposed by Oleh Faynshteyn, Leipzig, Germany

Let a, b and c be the side-lengths, and s the semi-perimeter of a triangle. Show that

$$\frac{a^2 + b^2}{(s - c)^2} + \frac{b^2 + c^2}{(s - a)^2} + \frac{c^2 + a^2}{(s - b)^2} \ge 24.$$

• 5341: Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain

Let z_1, z_2, \dots, z_n , and w_1, w_2, \dots, w_n be sequences of complex numbers. Prove that

$$Re\left(\sum_{k=1}^{n} z_k w_k\right) \le \frac{3}{(n+1)(n+2)} \sum_{k=1}^{n} |z_k|^2 + \frac{3n^2 + 6n + 1}{20} \sum_{k=1}^{n} |w_k|^2.$$