W42. Let $a, b, c > 0$ and $a + b + c = 1$.Then

$(a + 2ab + 2ac + bc)^a (b + 2bc + 2ba + ca)^b (c + 2ca + 2cb + ab)^c \le 1$. (1)

Marius Drăgan.

Solution by Arkady Alt, **San Jose**, **California**, **USA.**

By weighted AM-GM Inequality with weights (a, b, c) we obtain

$$\prod_{cyc}(a + 2ab + 2ac + bc)^a \le \sum_{cyc} a \cdot (a + 2ab + 2ac + bc) = \sum_{cyc} a^2 + 2\sum_{cyc} a^2(b + c) + 3abc =$$

$$\sum_{cyc} a^2 + 2\sum_{cyc} ab(a + b) + 3abc = \sum_{cyc} a^2 + 2\sum_{cyc} ab(a + b + c) - 3abc =$$

$$\sum_{cyc} a^2 + 2(a + b + c)(ab + bc + ca) - 3abc = (a + b + c)^2 - 3abc = 1 - 3abc \le 1.$$

Equality in original inequality isn't holds but if instead it we consider inequality

$(a + 2ab + 2ac + 2bc)^a (b + 2bc + 2ba + 2ca)^b (c + 2ca + 2cb + 2ab)^c \le 1$

then by the same way we obtain

$$\prod_{cyc}(a + 2ab + 2ac + 2bc)^a \le \sum_{cyc} a^2 + 2\sum_{cyc} ab(a + b) + 6abc = (a + b + c)^2 = 1.$$