W36. Let $\triangle(x, y, z) = 2(xy + yz + zx) - (x^2 + y^2 + z^2)$ and let a, b, c be sidelengths of a triangle with area F. Prove that $\triangle(a^3, b^3, c^3) \leq \frac{64F^3}{\sqrt{3}}$.

Arkady Alt

W37. Let E be a inner Product Space with dot product $- \cdot -$ and F be proper nonzero subspace. Let $P : E \to E$ be orthogonal proiection E on F.
a). Prove that for any $x, y \in E$, holds inequality $|x \cdot y - xP(y) - yP(x)| \leq \|x\| \cdot \|y\|$
b). Determine all cases when equality occours

Arkady Alt

W38. Prove that $0 < \left(\frac{4^x+2^x+1}{x}\right)^x - 2^x < 1$ for all $x \in \left(0, \frac{1}{2e}\right]$.

Ionel Tudor

W39. Let $n \geq 2$ be a natural number and $a_i > 0$, $i = \overline{1, n}$. If $S = \sum_{i=1}^n a_i$ and $x_i = S - a_i$, then the following inequality holds:

$$\frac{\prod_{i=1}^n \sqrt{a_i}}{\sqrt[(n-1)]{\prod_{1 \leq i < j \leq n}(a_i + a_j)}} \leq \frac{\prod_{i=1}^n \sqrt{x_i}}{\sqrt[(n-1)]{\prod_{1 \leq i < j \leq n}(x_i + x_j)}}.$$

Ovidiu Bagdasar

W40. Prove that if $x_i > 0$, $i = \overline{1, n}$, then the next inequality holds:

$$\sum_{i=1}^{n} \frac{S_{\alpha+\beta} - x_i^{\alpha+\beta}}{S_\alpha - x_i^\alpha} \leq n \cdot \frac{S_{\alpha+\beta}}{S_\alpha}, \tag{1}$$

provided that $\alpha\beta \geq 0$ and $S_p = \sum_{i=1}^n x_i^p$, for any real number p.

Ovidiu Bagdasar

W41. Let $n \geq 2$ a natural number and the numbers $a_i > 1$, $i = \overline{1, n}$. Prove that

$$\sum_{i=1}^{n} \frac{\log_{a_i} a_{i+1}^{n-1}}{S - a_i} \geq \frac{n^2}{\sum_{i=1}^n a_i}.$$

We consider that $a_{n+1} = a_1$, and $S = \sum\limits_{i=1}^{n} a_i$.

Ovidiu Bagdasar

W42. Let ABC be an acute triangle. The angle bisectors from A, B, C meet the opposite sides in A_1, B_1, C_1, respectively. Let R and r be the circumradius and the inradius of the triangle ABC, respectively. Let R_A, R_B, and R_C the circumradii of the triangles AC_1B_1, BA_1C_1, and CB_1A_1, respectively. Prove that

$$R_A + R_B + R_C \geq R + r.$$

Pál Péter Dályay

W43. Let f be a continuous real function defined on the set of the nonnegative real numbers for which the following integrals are convergent: $S = \int_0^\infty f^2(x)dx$, $T = \int_0^\infty xf^2(x)dx$, $U = \int_0^\infty x^2f^2(x)dx$. Prove that