W34. For any real $a, b, c, d > 0$ prove inequality

$$\frac{a}{\sqrt[3]{a^3+63bcd}}+\frac{b}{\sqrt[3]{b^3+63acd}}+\frac{c}{\sqrt[3]{c^3+63abd}}+\frac{d}{\sqrt[3]{d^3+63abc}} \geq 1.$$

Arkady Alt

W35. For any positive a, b, c such that $a+b+c=ab+bc+ca$ prove that

$$\frac{2}{3}\left(a^3+b^3+c^3\right) \geq abc+\frac{a^4+b^4+c^4}{a^2+b^2+c^2}.$$

Arkady Alt

W36. a). If $a_1, a_2, a_3, , , , a_{n+1}, n \geq 2$ are positive real numbers that satisfy inequality

$$\left(a_1^2+a_2^2+\ldots+a_{n+1}^2\right)^2 > n\left(a_1^4+a_2^4+\ldots+a_{n+1}^4\right)$$

then any n of them, let it be $a_1, a_2, a_3, , , , a_n$, satisfies inequality

$$\left(a_1^2+a_2^2+\ldots+a_n^2\right)^2 > (n-1)\left(a_1^4+a_2^4+\ldots+a_n^4\right)$$

b). If $a_1, a_2, a_3, , , , a_{n+1}, n \geq 2$ are positive real numbers that satisfy

$$\left(a_1^2+a_2^2+\ldots+a_{n+1}^2\right)^2 > n\left(a_1^4+a_2^4+\ldots+a_{n+1}^4\right)$$

then for any $1 \leq k_1 < k_2 < k_3 \leq n+1$ holds inequality

$$\left(a_{k_1}^2+a_{k_2}^2+a_{k_3}^2\right)^2 > 2\left(a_{k_1}^4+a_{k_2}^4+a_{k_3}^4\right)$$

that is $a_{k_1}, a_{k_2}, a_{k_3}$ be side lengths of some triangle.

Arkady Alt

W37. Let ABC be a non-obtuse angles triangle with usual notations. Prove that

$$\frac{2r}{R}+\frac{r^2}{R^2} \leq \sum \frac{a^2 \cos A}{bc} \leq 1+\frac{r}{R}.$$

Arkady Alt