PP48051. Prove that for real $a, b, c, k > 0$ such that $a + b + c = 3$, $k \leq 1$ and any $n \in \mathbb{N} \setminus \{1\}$ holds inequality

$$2\left(a^n + b^n + c^n\right) + 3\left(abc\right)^k \geq 9.$$

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PP48052. Find $\max\left\{a^{p+3}b^p + a^p b^{p+3} \mid a, b \geq 0 \text{ and } a + b = 1\right\}$, $p \in \mathbb{R}^+$

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PP48053. Prove that for any $x \in [0, 1]$ holds inequality $e^x + e^{\sqrt{1-x^2}} \geq e + 1$.

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PP48054. Find
$$\max\left\{\sum_{k=1}^{n}\frac{1}{1+x_k^2} \mid x_1, x_2, ..., x_n \in (-\infty, 1] \text{ and } \sum_{k=1}^{n} x_k = 1\right\}, n \geq 3$$

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PP48055. Prove that $\left\lfloor \left(\sqrt{a+1}+\sqrt{a}\right)^{2n}\right\rfloor - 1$ divisible by $4a$ for any $n \in \mathbb{N}$.

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PP48056. Find the limit $\lim\limits_{n\to\infty} n\left(\left(\frac{\sqrt[n]{t}+1}{2}\right)^n - \sqrt{t}\right)$

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PP48057. Let a, b, c be positive real numbers such that $a+b+c=1$. Prove that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+2 \geq \sqrt{13+\frac{4}{abc}}.$$

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