PP39670. Let $n > 0$ be an integer. Calculate

$$\lim_{x\to 0}\left((2n+1)!\frac{\sinh x - x - \frac{x^3}{3!} - \cdots - \frac{x^{2n-1}}{(2n-1)!}}{x^{2n+1}}\right)^{\frac{1}{x^2}}.$$

Ángel Plaza

PP39671. Let $x, y \geq 1$. Prove that

$$\frac{xy\,(xy-1)\,(x+y)}{\left(x^2+1\right)^2\left(y^2+1\right)^2} \leq \frac{3\sqrt{3}}{64}$$

or find

$$\max_{x,y\geq 1}\frac{xy\,(xy-1)\,(x+y)}{\left(x^2+1\right)^2\left(y^2+1\right)^2}.$$

Arkady M. Alt

PP39672. Prove that in any acute triangle holds inequality

$$\sum\sqrt{\frac{\cos B\cos C}{\cos\left(A-B\right)\cos\left(C-A\right)}} \leq \frac{3}{2}\,.$$

Arkady M. Alt

PP39673. Calculate

$$\sum_{k=1}^{\infty}\frac{1}{4^{k-1}k^2}\binom{2k-2}{k-1}.$$

Arkady M. Alt

PP39674. For any real $a > 0$ and any natural $n \geq 2$ prove that

$$\prod_{k=1}^{n}\left(1+\frac{a}{\sin^2\frac{k\pi}{2n}}\right) \geq (1+a)\,(1+2a)^{n-1}\,.$$

Arkady M. Alt

PP39675. Prove that in any triangle with circum radius R and inradius r holds inequality

$$\left(a^2+b^2+c^2\right)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\geq \frac{2\left(5R^2-2r^2\right)}{R^2}.$$

Arkady M. Alt

PP39676. For any positive a, b, c prove that

$$\sum \frac{a}{b+c}\cdot\sum\frac{a}{7a+b+c}\geq\frac{1}{2}.$$

Arkady M. Alt

PP39677. Prove that in any acute angled triangle with circumradius R holds inequality

$$\left(a^2+b^2+c^2\right)\left(\frac{1}{2R^2}+\frac{a^2}{b^2+c^2}+\frac{b^2}{c^2+a^2}+\frac{c^2}{a^2+b^2}\right)\leq 9.$$

Arkady M. Alt

PP39678. Let l_a, l_b, l_c and F be, respectively, lengths of angle bisectors and area of a triangle ABC. Prove that

$$\frac{1}{\sqrt{3}}\min\left\{l_a^2, l_b^2, l_c^2\right\}\leq F\leq\frac{1}{\sqrt{3}}\max\left\{l_al_b, l_bl_c, l_cl_a\right\}$$

Arkady M. Alt

PP39679. Let ABC be an equilateral triangle with circumradius R and let P be a point inside the triangle. Prove that for side lengths a_p, b_p, c_p of pedal triangle $A_pB_pC_p$ holds inequality

$$a_pb_pc_p\leq\frac{27\sqrt{3}}{64}R^3.$$

Arkady M. Alt

PP39680. Let $f:[0,1]\to[1,\infty)$ be continuous on $[0,1]$ and differentiable on $(0,1)$ function and let $p_n=\prod_{k=1}^{n-1} f\left(\frac{k}{n}\right)$. Calculate:

a) $\lim_{n\to\infty} \frac{\sqrt[n+1]{p_{n+1}^n}}{p_n}$. b) $\lim_{n\to\infty} \frac{p_{n+1}}{p_n}$.

Arkady M. Alt

PP39681. For any interior point P in a triangle ABC let A_p, B_p, C_p feets of perpendiculars from P to sides BC, CA, AB ,respectively, and let $a_p := B_pC_p, b_p := C_pA_p, c_p := A_pB_p$ (side lengths of Pedal triangle $A_pB_pC_p$). Prove that:

a) $\frac{a_p}{a^2} + \frac{b_p}{b^2} + \frac{c_p}{c^2} \geq \frac{\sqrt{3}}{2R}$. b) $\frac{a_p^2}{a^4} + \frac{b_p^2}{b^4} + \frac{c_p^2}{c^4} \geq \frac{1}{4R^2}$.

Arkady M. Alt

PP39682. Let r the radius of the sphere be inscribed in a tetrahedron. Show that we have inequality:

$$\frac{S_1}{S_2S_3S_4(S_4+S_1)} + \frac{S_2}{S_3S_4S_1(S_1+S_2)} +$$

$$+\frac{S_3}{S_4S_1S_2(S_2+S_3)} + \frac{S_4}{S_1S_2S_3(S_3+S_4)} \geq \frac{128r^3}{27V^3},$$

where S_1, S_2, S_3, S_4 are the areas of the tetrahedral faces, r is the radius of the sphere inscribed in the tetrahedron, and V is the volume of the tetrahedron.

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