We have that $S_2 = \frac{\pi^2}{48} + \frac{\ln 2}{2} - \ln^2 2$ (see [?, Problem 3.28, p. 143]).

Open problem. Calculate S_k for $k \geq 3$.

(b) An alternating quadratic series.

$$A_k = \sum_{n=1}^{\infty} (-1)^k \left(\ln k - \frac{1}{n+1} - \frac{1}{n+2} - \cdots - \frac{1}{kn} \right)^2.$$

We have $A_2 = \frac{\pi^2}{48} - \frac{\pi \ln 2}{8} - \frac{7}{8} \ln^2 2 + \frac{G}{2}$ (see [?, Problem 3.44, p. 146]).

Open problem. Calculate A_k for $k \geq 3$.

REFERENCE

[1] O. Furdui.: Limits, Series and Fractional Part Integrals. Problems in Mathematical Analysis, Springer, New York, 2013

Ovidiu Furdui

OQ5594. Find

$$\lim_{n \to \infty} n^{(2m-1)/2} \cdot \Delta^m \left(\sqrt{n} \right),$$

where Δ^m is difference operator of the m-th order.
Conjecture:

$$\lim_{n \to \infty} n^{(2m-1)/2} \cdot \Delta^m \left(\sqrt{n} \right) = \frac{(-1)^{m+1} m!}{2^{m+1} \sqrt{n(n+1) \ldots (n+2m)}}$$

Arkady M. Alt

OQ5595. Let $(e_n)_{n \in \mathbb{N} \cup \{0\}}$ be sequence defined by recurrence

$$e_{n+1} = \frac{1}{n+1} \sum_{k=0}^{n} \frac{k+1}{k+2} \cdot e_{n-k}$$

and $e_0 = 1$. Prove that $e_n < e_{n+1}$ for any $n \in \mathbb{N} \cup \{0\}$.

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