MATHEMATICS OF NATURE AND NATURE OF MATHEMATICS
(AMARNATH MURTHY)

"If people do not believe that mathematics is simple, **it is only because they do not realize** ***how complicated life is "***
(John Von Neumann)

Abstract

It is a common perception that mathematics is a dull subject with no connection to life. Also there is a myth that mathematics is a subject of study not for ordinary people rather for some abnormal, eccentric, serious, absent minded, fat, bald and bespectacled people with wrinkles on their foreheads who lead little or no social life. On the contrary Mathematics is quite interesting and is present in many aspects of life and anybody can study and understand mathematics if one believes in oneself.

Mathematics has been thought of as a left cortical (brain)" subject, and identified as 'linear', analytical, logical, grey, cold, hard, complex, difficult, and BORING. In fact mathematics is a whole cortical-skill-subject that involves words, lines, logic, patterns, symmetry, rhythm, space, association and the over-riding concept of fun.

The author endeavors to justify his claim in the following pages.

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Scientists do not study nature just because it is useful: they study nature because it is beautiful. If nature were not beautiful, it would not be worth knowing. And if nature were not worth knowing, life would not be worth living. We cannot understand (nature) if we do not first learn the language and grasp the symbols in which it is written. Mathematics does not only describe nature, it is **nature's language.** There is nothing in our lives, in our world, in our universe, that cannot be expressed with mathematical theories, numbers, and formulae. **Mathematics is the key to understanding our world around us. It is perhaps the purest of the pure mental endeavor of humankind.** Mathematics has been called the mother of all sciences; to me it is the backbone of all systems of knowledge.

Mathematics is a tool that has been used by man for many years. It is a key that can unlock many doors and show the way to different logical answers to seemingly impossible problems. Not only can it solve equations and problems in everyday life, but it can also express quantities and values precisely with no question or room for other interpretation. There is no room for subjectivity. Though there is a lot of mathematics in politics, there is no room for politics in mathematics. Coming from a powerful leader two + two can not become five, it will remain four.

Mathematics is not fundamentally empirical —it does not rely on sensory observation or instrumental measurement to determine what is true. **Indeed, mathematical objects themselves cannot be observed at all!**

Richard Feynman. Says: "To those who do not know Mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty of nature. ... If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in."

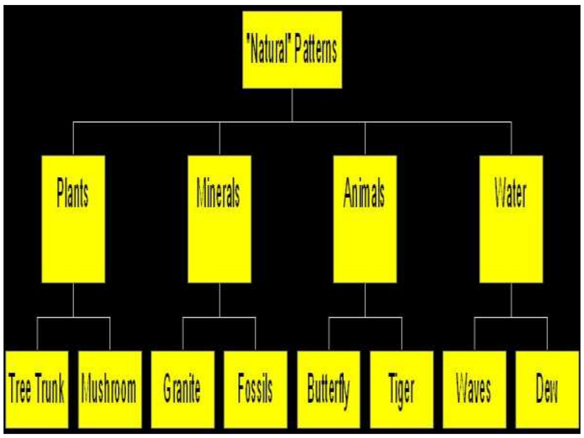
Mathematics is obviously the most interesting, exciting, and useful, challenging, satisfying, inspiring, impressive, consistent, stimulating and beautiful subject in existence! It is the science of patterns and order and the study of measurement, properties, and the relationships of quantities; using numbers and symbols.

"Mathematics, rightly viewed, possesses not only truth, but supreme beauty.'' **--- Bertrand Russell**

Mathematics and Nature

Mathematics is such a beautiful game of numbers, notions and notations designed by man that even **nature behaves mathematically**.

Nature is full of beautiful patterns: Bubbles, Waves, Bodies, Branches, Breakdowns, Fluids, Grains, and Communities etc. Patterns are result of naturally occurring processes. One of the purposes of natural science is to build models of these processes, **the complexity arises from simplicity.** From rainbows, river meanders, and shadows to spider webs, honeycombs, and the markings on animal coats, all these patterns can be described mathematically.

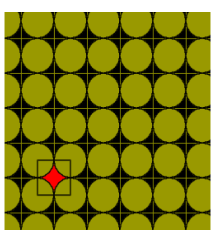


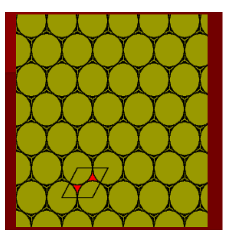
Fact of creation: Some living beings which even do not have a brain perfectly perform so complicated tasks as not to be accomplished even by human beings.

The honeycomb is hexagonal in shape because a hexagon is the most appropriate geometric form for the maximum use of a given area and also the hexagonal structure provides the maximum strength. The honey bees need not go to a school to learn this.

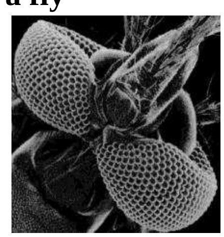
Area of a single cell (square units)

equilateral triangle	square	hexagon
0.048	**0.063**	**0.075**





Compound eye of a fly



A hexagonal cell requires the **<u>minimum amount of wax</u>** for construction while it **<u>stores the maximum amount of honey.</u>** The red space between cells depicts the wasteful area for a square and a hexagonal construction. A hexagonal structure also provides the maximum strength. Although the wax cell walls may be only about **0.05mm thick**, each cell can support **25 times** its own weight.

Considering that for each gram of wax produced the bee needs to consume 6 - 7 grams of honey, it is to the bees' advantage that the shape providing the maximum area has the minimum expenditure of materials and energy.

The method used in the construction of the honeycomb is also very amazing: bees start the construction of the hive from two-three different places and weave the honeycomb simultaneously in two-three strings. Though they start from different places, the bees, great in number, construct identical hexagons and then weave the honeycomb by combining these together and meeting in the middle. The junction points of the hexagons are assembled so deftly that there is no sign of their being subsequently combined.

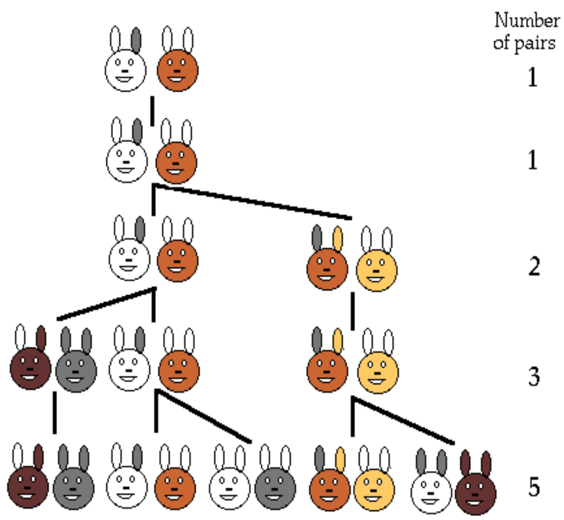
The **compound eyes of insects** typically exhibit **hexagonal packing schemes**. No doubt the same criteria of maximizing light-sensitive area coverage while minimizing the volume of inert edge-cell material that are familiar from honeycombs apply here.

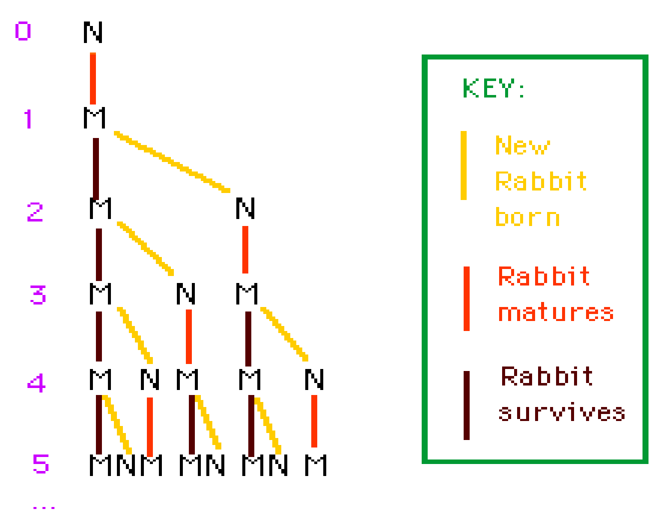
Patterns can be constructed by **distributed intelligence:** Termites build castles with similar complexity to human buildings. There is no plan, there is no blueprint. No foreman organizes the work of workers and no one governs the workers. The termite mound is built by self organized efforts.

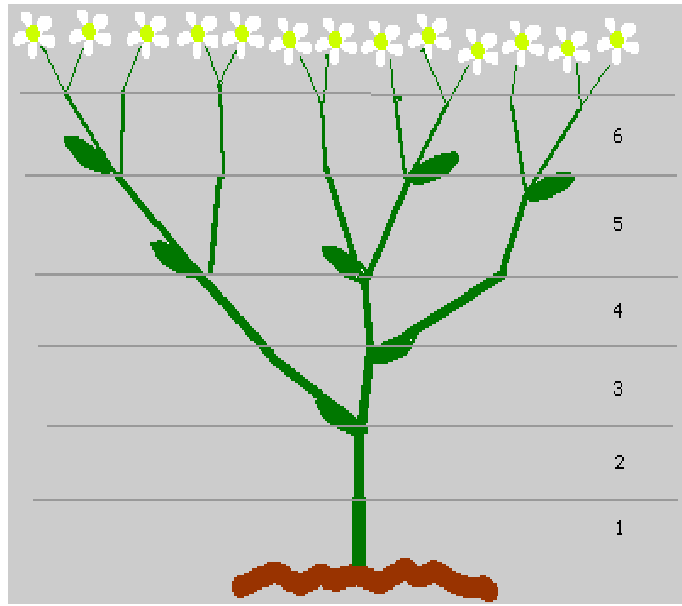
The Fibonacci numbers are Nature's numbering system. (Fibonacci was an Italian mathematician of 13th century who introduced the decimal system of INDIA in Europe). The Natural sequence is formed by adding a succeeding number to the previous root number, and is as follows: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 **…** and so on. In other words 5+8=13, 8+13=21, and 21+13=34 and so on. If one divides a term of the sequence by the previous one, one gets a ratio, called golden ratio **(divine proportion),** 144/89 = **1.618**. This ratios approaches phi, (the golden number) an irrational number. They appear everywhere in nature, from the leaf arrangement in plants, to the pattern of the florets of a flower, the bracts of a pinecone, or the scales of a pineapple.

The original problem that Fibonacci investigated (in the year 1202) was about how fast rabbits could breed in ideal circumstances.

He has a newly-born pair of rabbits, one male, one female, and he puts them in a BIG cage. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits. Suppose that our rabbits never die and that the female always produces one new pair (one male, one female) every month from the second month on. The puzzle that Fibonacci posed was... **How many pairs will there be in one year?**

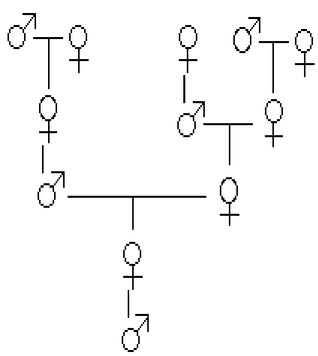






One plant shows the Fibonacci numbers in **"growing points"** that it has. Suppose that when a plant puts out a **new shoot**, that shoot has to grow two months before it is strong enough to support branching. If it branches every month after that at the growing point, we get the picture shown above.

Honeybees, Fibonacci numbers and Family trees



The genealogy of the bee is a pattern. This pattern is the result of pathogenesis, the development of an unfertilized egg into an adult animal without fusion with sperm. The queen bee mates only once. She can then produce either unfertilized eggs or fertilized eggs. The unfertilized eggs become male drones, while the fertilized eggs become female workers or queens. In other words, a female bee has two parents, and a male bee has only one parent; a female. Female bees reproduce by parthenogenesis during the spring and summer. In the fall, the eggs produce both males and females. These insects mate, and the females produce fertilized eggs that hatch in the spring.

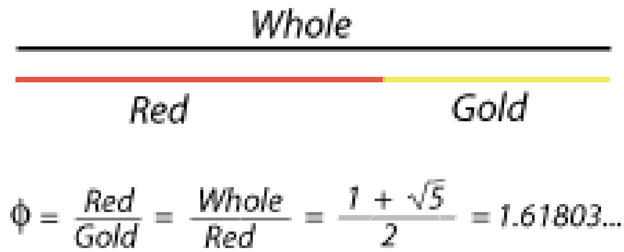
Family Tree of Drone Bee

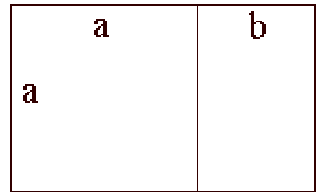
- He had 1 parent, a female.
- He has 2 grand-parents, since his mother had two parents, a male and a female.
- He has 3 great-grand-parents: his grand-mother had two parents but his grand-father had only one.

The total of all the males and the total of all the females that make each generation forms an overlapping Fibonacci series repeated twice; One for males and one for females

Number of	parents:	grand-parents:	great-grand-parents:	great, great grand parents:	gt,gt,gt grand parents:
of a MALE bee:	1	2	3	5	8
of a FEMALE bee:	2	3	5	8	13

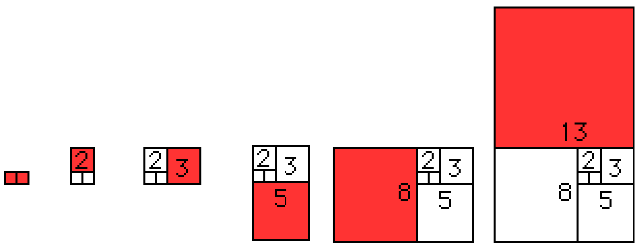
Golden Ratio (Divine Proportion)/Golden Rectangle.

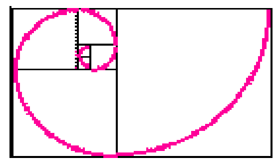




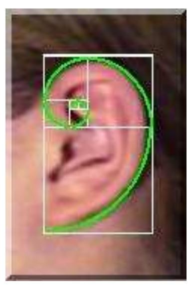
a/b = (a+b)/a

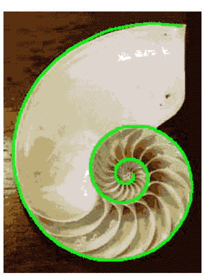
A **golden rectangle** is one which can be divided into a square and a similar rectangle.
A Golden spiral is formed on a golden rectangle as below**.**

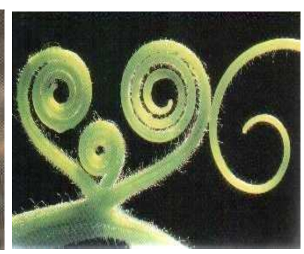


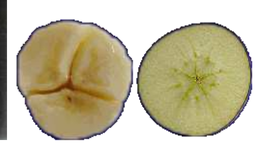


$R = R_0 e^{\theta/\theta_0}$



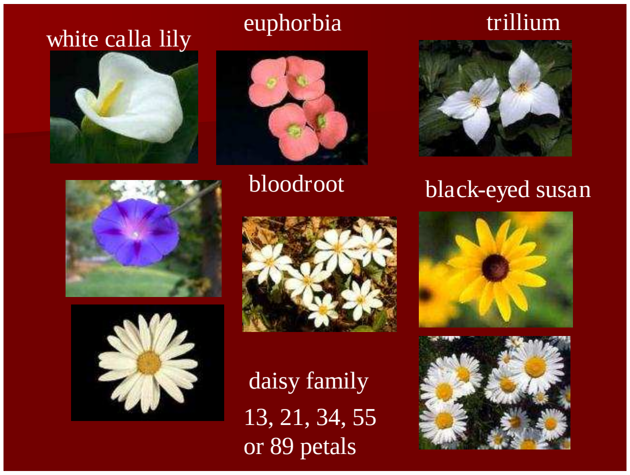


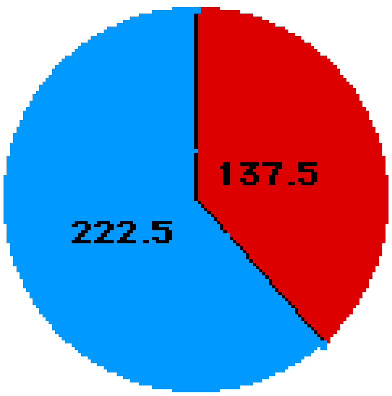




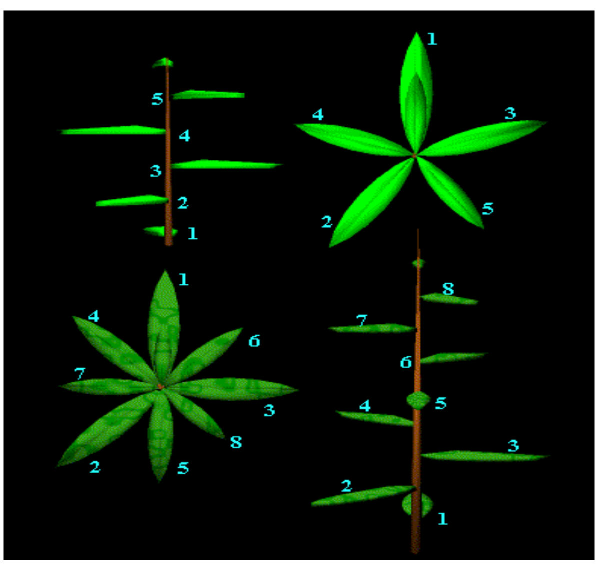
1. Take any two numbers at random, second bigger than the first (such as 214 and 346)
2. Add them together
3. Add the result to the second largest number
4. Repeat from step 2 for a while (five or six times, maybe)
5. Divide the last result you got by the second-to-last result

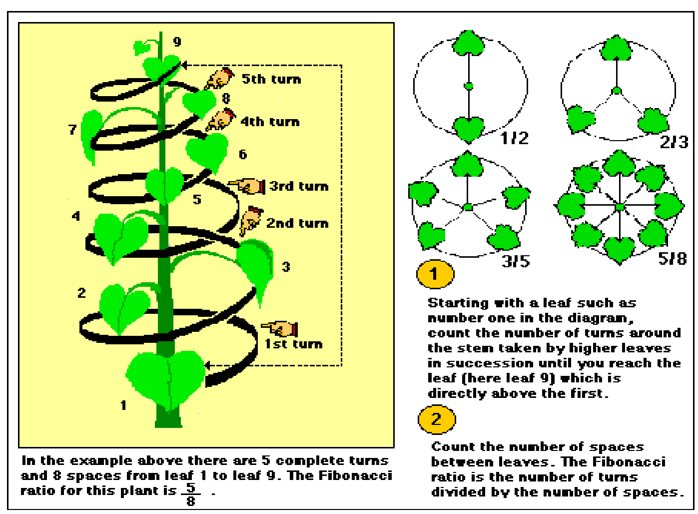
I'll bet your answer is somewhere close to 1.618 that is phi.

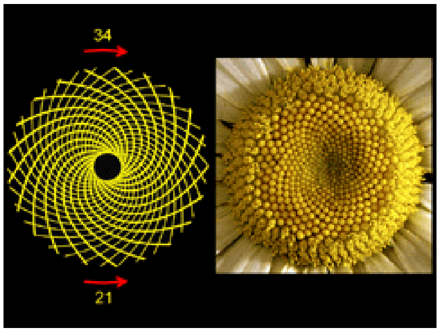


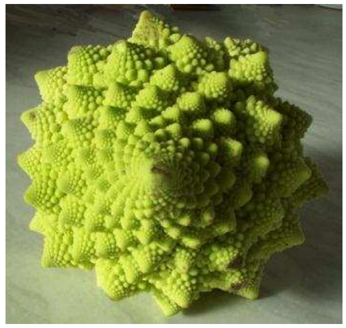


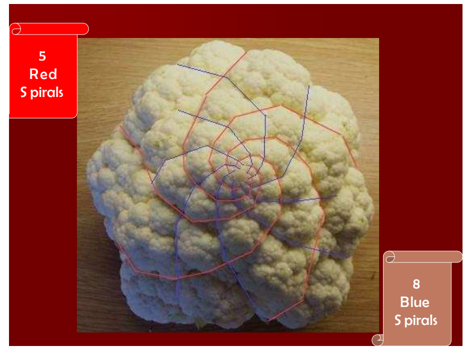
ARRANGEMENT OF LEAVES, PETALS, SEEDS IS 0·382 x 360 = 137·5°.



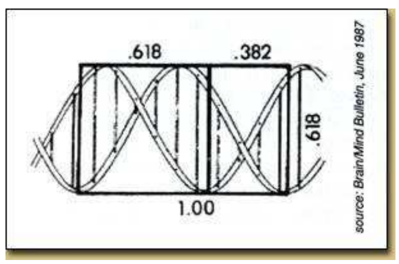


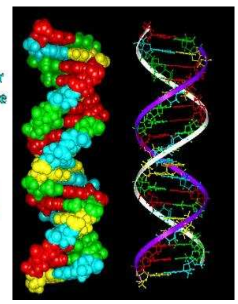






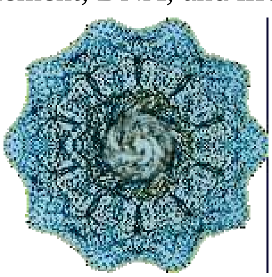
These patterns are not only beautiful rather we have the best packing so that each leaf gets the **maximum exposure to light,** casting the least shadow on the others. This also gives **the best possible area exposed to falling rain** so the rain is directed back along the leaf and down the stem to the roots. For flowers or petals, it gives the **best possible exposure to insects to attract them for pollination. Nature uses spirals to prevent overcrowding. The Fibonacci numbers, golden ratio**(or divine proportion)**,** and **the golden spiral** are therefore applicable to the growth of every living thing, including a single cell, a grain of wheat, a hive of bees, sea shell, plants, and even all of mankind.

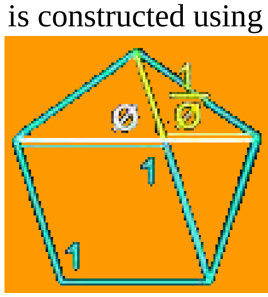


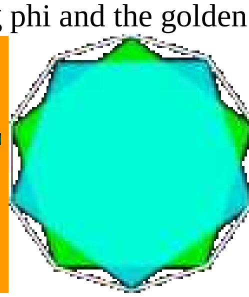


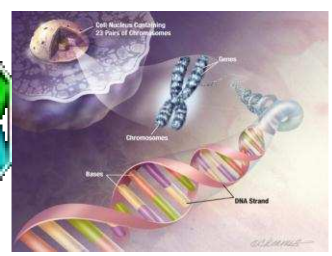
The DNA molecule, the program for all life, is based on the Golden section. It measures **34 angstroms** long by **21 angstroms** wide for each full cycle of its double helix spiral. 34 and 21, of course, are numbers in the Fibonacci series and their ratio, 1.6190476 closely approximates Phi, 1.6180339.

The **DNA cross-section** is also based on Phi. A cross-sectional view from the top of the DNA double helix forms a decagon: A decagon is in essence two pentagons, with one rotated by 36 degrees from the other, so each spiral of the double helix must trace out the shape of a pentagon. The ratio of the diagonal of a pentagon to its side is Φ to 1. So, no matter which way you look at it, even in its smallest element, DNA, and life, is constructed using phi and the golden section!

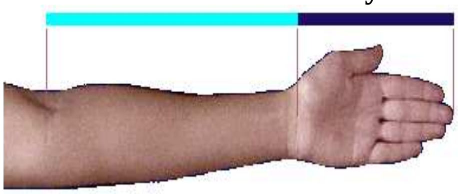


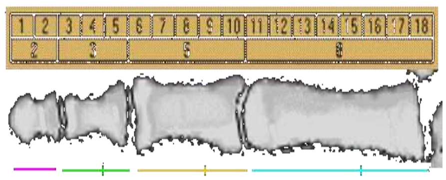


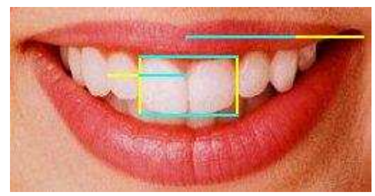


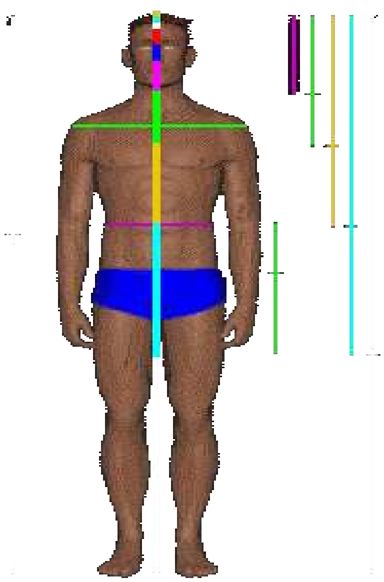


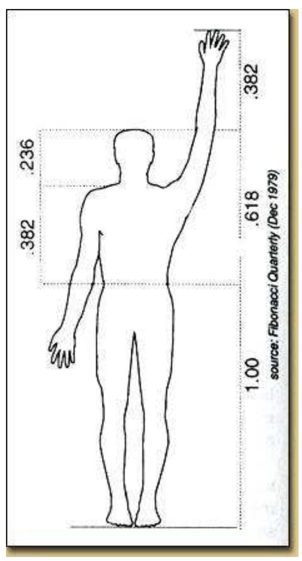
The human body is based on Phi and the number 5.

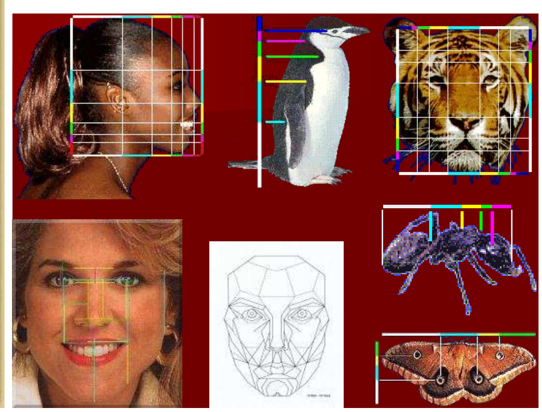




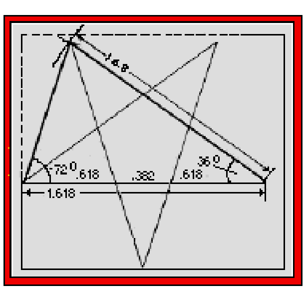


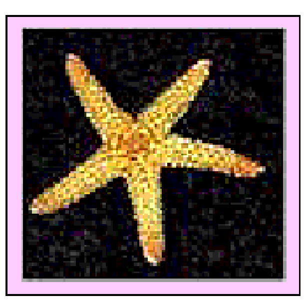


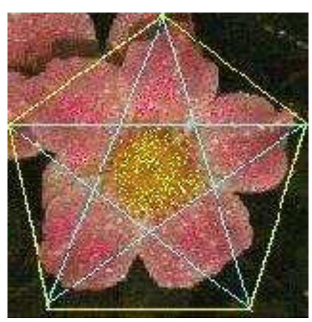




When the distance between the navel and the foot is taken as 1 unit, the height of a human being is equivalent to 1.618.





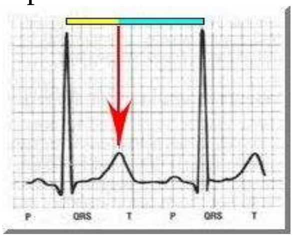


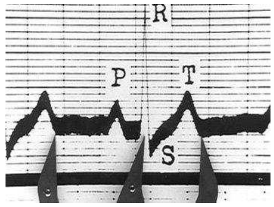
The Sarva Dharm Symbol is based on pentagon.

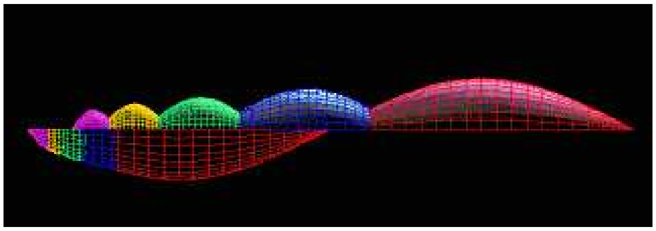
The golden caliper: the arms move so that the ratio of the two parts is always maintained 1.618… = phi.

The Human Heartbeat : A peaceful heartbeat is said to beat in a Phi rhythm.

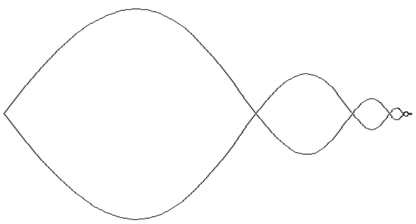
While heartbeats vary, some believe that a heartbeat that reflects this perfect phi relationship represents a state of being that is one of **health, peace and harmony**.







Perfect breathing



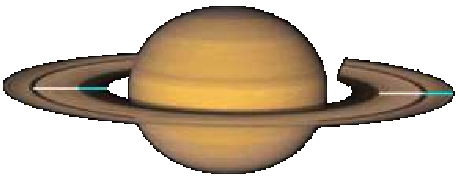
The breath moves in and out in the geometry of perfect damping, or the perfect way to approach the icy stillness of oneness. The depth and the duration of each adjacent breath get smaller by ratio PHI (golden mean).

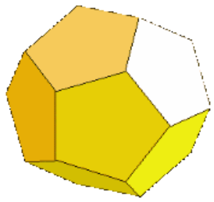
Phi appears in the Solar System and The Universe

From the distances between the planets, to the structure of Saturn's rings to the shape of the Universe itself, phi is found again and again in different manifestations. New findings reveal that the universe itself is in the shape of a dodecahedron, a twelve-sided geometric solid with pentagon faces, all based on phi.

Saturn's magnificent rings show a division at a golden section of the width of the rings.

Curiously, even the relative distances of the ten planets and the largest asteroid average to **phi.** There's even an unusual energy source at the frequency of phi that is found in the universe.





The cycle of the moon resembles that of the digit sequences in phi behind the decimal point as you multiply phi with itself. These digit sequences are the same for the odd powers of phi and their reciprocals in the bold rows below, but not for the even powers between them.

Digit sequence symmetries in positive and negative powers of phi:

phi^n	$1 / phi^n$
phi^1 = 1.61803398875	**$1/phi^1$ = 0.61803339887**
phi^2 = 2.61803398875	$1/phi^2$ = 0.38196601125
phi^3 = 4.23606797750	**$1/phi^3$ = 0.23606797750**

Planetary orbits in our solar system

The time span of planetary orbits in our solar system (i.e., the time it takes for each planet to make one complete revolution around the sun), are related by Fibonacci relationships. Specifically, given that the Earth takes one year to orbit the sun, the time that it takes Venus to orbit the sun is 1.618 to the -1 years. The time it takes Mercury to orbit the sun is approximately 1.618 to the -3 years. Mars is roughly 1.618 years, the asteroid belt is roughly 1.618 to the 3rd years, Jupiter is approximately 1.618 to the 5th years, so on and so forth (note that Pluto does not fit).

Fibonacci and music: Fibonacci and phi relationships are often found in the timing of musical compositions. As an example, the **climax** of songs is often found at roughly **the phi point (61.8%)** of the song, as opposed to the middle or end of the song.

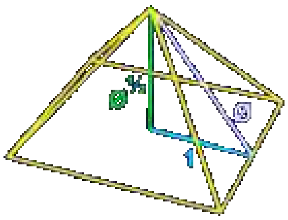
Periods of man's life

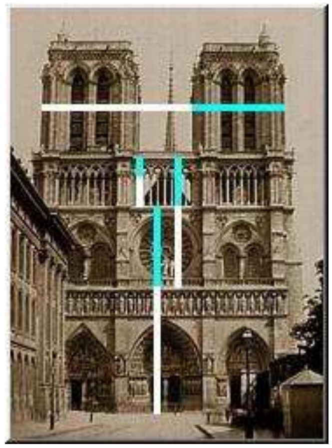
Man's ages correspond to the following years: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, and all man's life can be divided into 7 periods: about one year corresponds to infancy, 1-8 years to **childhood**, 8-13 to **adolescence,** 13-21 to **youth**, 21-34 years to **second youth,** 34-55 years to **maturity,** 55-89 years to **old age**.

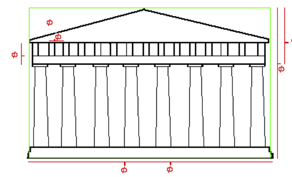
The Design Of Life: Human Development From A Natural Perspective

Human Age	Development Stage	Key Attributes
0	Gestation	Conception
1	Newborn	Birth
1	Infant	Walking, vocalizing
2	Toddler	Talking, expressing, imitating
3	Toddler	Self image and control, toilet training
5	Early child	Formal education begins
8	Mid child	Age of reason, knowing of right and wrong
13	Adolescent	Thinking, puberty, sexual maturation and drive
21	Young adult	Full physical growth, adult in society, education complete, beginning career, financial responsibility, eligible for voting
34	Mid adult	Refinement of adult skills, parenting role
55	Elder adult	Fulfillment of adult skills, serving, retirement begins with eligibility for Medicare, Social Security and AARP
89	Completion	Insight and wisdom into life

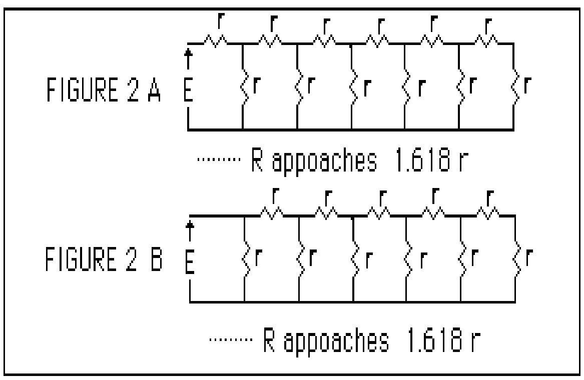
Golden ratio in ancient art







In electrical engineering the Fibonacci numbers occur. For example, if a series unit resistances are connected alternately in series and in parallel, as shown in **Figure 2 A** the resistances of the entire circuit is expressed by the continued fraction:



$$R = r + \cfrac{1}{\cfrac{1}{r} + \cfrac{1}{r} + \cfrac{1}{\cfrac{1}{r} + \cfrac{1}{r} + \cfrac{1}{\cfrac{1}{r} + \cdots\cdots}}}$$

$$\frac{R}{r} = 1, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, \frac{13}{21}, \frac{21}{34}, \cdots\cdots\ 0.618$$

In accordance with the Fibonacci series., if the first resistance in the circuit is in parallel instead of series, as in figure 2 B, the initial term in the equation for R is omitted and successive values for R/r become: **.618…**

Phi appears in quantum mechanics as well.

Fibonacci Numbers in Finance

Our brains are hardwired to find Fibonacci numbers naturally pleasing. the fibs" are supposed to predict the behavior of the stock market. The Fibonacci Studies and Finance When used in technical analysis, the golden ratio is typically translated into three percentages: – 38.2%, 50% and 61.8%. Changes in stock prices largely reflect human opinions, valuations and expectations. A study by mathematical psychologist Vladimir Lefebvre demonstrated that humans exhibit positive and negative evaluations of the opinions they hold in a ratio that approaches phi, with 61.8% positive and 38.2% negative.

Some ϕ mathematics :

ϕ as continued fraction

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\cdots}}}}$$

ϕ as repeated square root

$$\phi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}$$

ϕ and e

$$e = \frac{\sum_{k=0}^{\infty} \frac{F(n+k)}{k!}}{\sum_{k=0}^{\infty} \frac{F(n-k)}{k!}}$$

π, ϕ and Fibonacci numbers.

$$\frac{\pi}{4} = arctg(phi) + arctg(phi^3)$$

$$\frac{\pi}{4} = arctg\frac{1}{2} + \sum_{k=1}^{\infty}(-1)^{k+1} arctg\frac{1}{F(2k+1)}$$

2cos (π/5) = ϕ

Pi/4 = arctan(1/2) + arctan(1/5) + arctan(1/13) + arctan(1/34) + arctan(55) +...

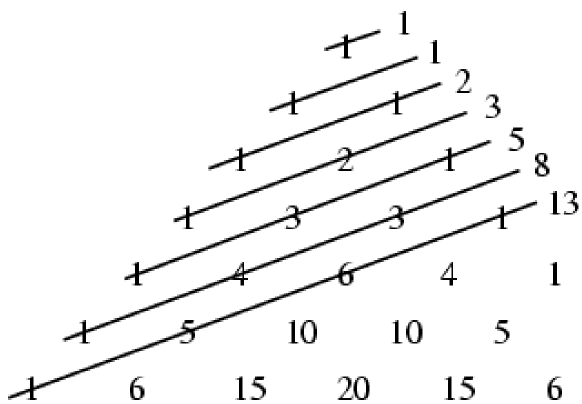
f(n) = Φ^n / 5½ (This provides an estimate which always rounds to the correct Fibonacci number.). $\phi^2 = \phi + 1$, $1/\phi = .618\ldots = \phi - 1$.

$1/phi + 1/phi^2 + 1/phi^3 + 1/phi^4 + 1/phi^5 + 1/phi^6 + \ldots = phi$

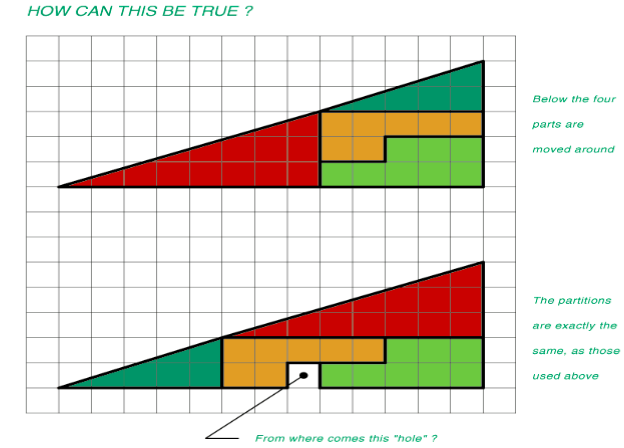
$x/(1-x-x^2)$ = SUM $F(n)*x^n$

$$
\begin{array}{r}
x + x^2 + 2x^3 + 3x^4 + \ldots \\
1 - x - x^2 \,\big)\, x \qquad\qquad\qquad \\
x - x^2 - x^3 \qquad\qquad \\
\hline
x^2 + x^3 \qquad\qquad \\
x^2 - x^3 - x^4 \qquad \\
\hline
2x^3 + x^4 \qquad \\
2x^3 - 2x^4 - 2x^5 \\
\hline
3x^4 + 2x^5
\end{array}
$$

PASCAL'S TRIANGLE



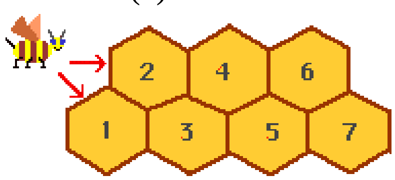
Fun with Fibonacci number problems and puzzles: There are many combinatorial problems (arrangements) which have Solutions as Fibonacci numbers



A closer look at the slanted sides of the trapezoidal and triangular pieces shows that they cannot be aligned as implied in the above fallacious illustrations. In fact, they are the **diagonals of two dissimilar rectangles of sizes 2x5 and 3x8 , respectively, and hence have distinct slopes. But the difference of the ratios (2/5 = .4 versus 3/8=.375) is too small to be perceived by the eye.** Note that the dissection cuts the sides of the squares according to the proportion 5:3. The illusion becomes even more effective if the numbers 3, 5, 8 are replaced by a triple of higher consecutive Fibonacci nos.

Making a bee-line with Fibonacci numbers

Here is a picture of a bee starting at the end of some cells in its hive. It can **start at either cell 1 or cell 2** and **moves only to the right** (that is, only to a cell with a higher number in it). There is only one path to cell 1, but two ways to reach cell 2: directly or via cell 1. For cell 3, it can go 123, 13, or 23, that is, there are three different paths. How many paths are there from the start to cell number n? it is Fibonacci (n)



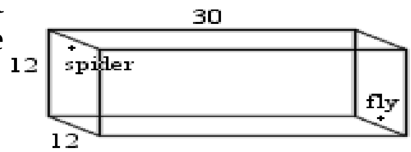
This ancient concept of the golden mean has been studied by artists, architects, mathematicians, musicians, philosophers and spiritual leaders throughout history. It has been proven to clearly manifest itself in the smallest particles of life such as the double-helix structure of DNA, as well as in plants, shells, insects, and the proportions of the human body. **It is the initial building block for the creation of beauty, form, function and order in the universe.**

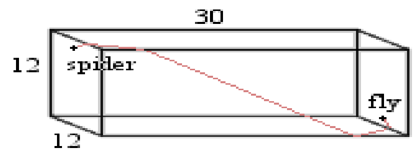
Animals using mathematics:

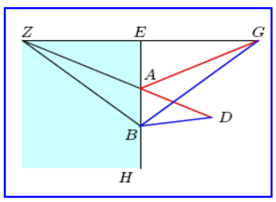
Animals don't do their thing **"using mathematics"** the way we do. Rather, natural selection, acting over millions of years, has equipped them with a range of physical and mental abilities that enable them to survive in their own evolutionary niche. A bird that navigates by the sun or the stars "solves" a problem in trigonometry only in the same way that a river flowing down hill "solves" a differential equation of fluid dynamics or the Solar System, by its very planetary motions, "solves" a particular instance of a many-body problem in gravitational dynamics (both feats that remain well beyond the capabilities of present-day mathematicians, incidentally). That is to say, it is only when the bird's activity is interpreted in human terms that the creature can be said to "solve a mathematics problem." The bird itself simply does what comes natural to it. As a result of our own evolutionary path, we human beings have found ways to be able to extend our own range of instinctive, unconscious behaviors so that we can mimic some of the activities of our fellow creatures.To many people, mathematics is merely a collection of techniques for calculating. But when you look around at many of the things we do with mathematics, you realize that it is a powerful mental framework that enables human beings to extend their capabilities well beyond those for which our evolution directly equipped us. According to the old joke, "If God had meant us to fly, he'd have given us wings." A more accurate version would be, "Obviously God wanted us to fly; that's why He gave us a brain that was capable of developing mathematics, which we could use to invent and build airplanes and develop methods and technologies to navigate when we are in the air."

In fact mathematics is a loom in which God weaves the nature."God is a Mathematician", so said Sir James Jeans**.**

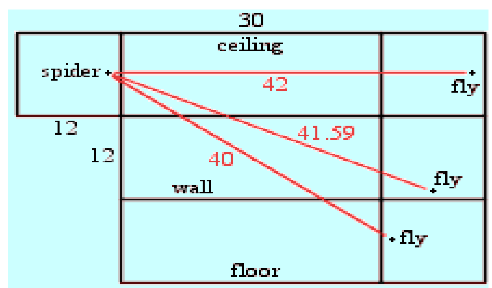
Light travels following the principle of the **fastest route or minimal travel-time** for the transition between two media. The straight line is the shortest route between two points, while the circle is the shortest periphery for a given area. The principle was then applied by Heron of Alexandria to prove that in a mirror, the angle of reflection is equal to the angle of incidence. Fifteen centuries later, Fermat (1601- 1665) used the same principle of the **fastest route or minimal travel-time** for the transition of light between two media, i.e. for the derivation of Snell's law.







The red path (GAD) is shorter than the blue path (GBD) where angle GAE = angle DAB



Animals also follow the **shortest path** to catch their prey while hunting. A spider on one wall of your drawing room follows the shortest path to reach its prey on the opposite wall via the ceiling. An engineer to locate and trace this path would rather require (to flatten up) opening up the walls to bring them to one plane, join the two points (spider and the prey) and then fold back to the original position.

Prime numbers:

Numbers **p** with only two factors 1 and **p** are called prime (2, 3, 5, 7, 11, 13,). All natural numbers are made of 1, prime and composite numbers. Prime numbers are nature's most important numbers because they are the "atoms" of mathematics, the hydrogen and oxygen of the world of numbers. Every number is built by multiplying prime numbers together. Despite 2,000 years of scrutiny, mathematicians are stumped by the task of predicting when the next prime will occur.

In nature, primes are the key to the survival of a strange species of insect. The prime-number cicadas hide in the ground for 17 years, and then emerge en masse from the earth into the forest. They sing loudly, eat, have sex, lay eggs and then die after six weeks of intensive partying. The forest goes quiet again for 17 years. For the cicadas, the primes aren't just some abstract curiosity, but the secret to their survival. But why did the cicadas choose 17, a prime number, for their hibernation? Scientists believe there is a predator that likes to crash their party and also emerges periodically after a certain number of years. **The cicadas found that by choosing a prime-number cycle for their party, they could keep out of step of the predator more often than if they'd chosen a non-prime such as 15.**

Conclusion:

The world to many of us seems to be a world of chaos, disorder, confusion, and above all, randomness. Indeed, the world has seemed this way for hundreds of thousands of years, the apparent randomness causing a catalyst for mankind to improve on what it was given. From naturally-growing wood we constructed houses, from stone we constructed forts, from petroleum we made plastic. All of evolution, all of history, can be summarized as an attempt by mankind to insulate itself from the harsh randomness of the outside world, an attempt to ensure the survival of the species by improvisation using our minds. What a comfort then it must have been to Fibonacci, Sir Isaac Newton, and other brilliant mathematicians to discover the underlying order in the universe, the one thing that bonds all random events together, from weather to erosion to growth and even life itself. As Dr. Stephen Marquardt so wisely stated, **"All life is biology. All biology is physiology. All physiology is chemistry. All chemistry is physics. All physics is math."(Marquardt, 2001) Math is indeed the underlying order of the universe, and the Fibonacci sequence and Phi are indeed some of the most fascinating discoveries made in the mathematical world since the time of Archimedes.**

The purpose of this article has been to remove some of the prevailing misconceptions about mathematics. The author endeavors to throw some light on the simplicity, beauty, utility, and fun aspects of this fascinating game of numbers, notions and notations and its role as a language of nature, so that today's children do not consider it as an inevitable evil. When you are not forced to learn mathematics your learning becomes easy.

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A sincere apology: The contents of this article are taken from tens of books and hundreds of web sites and it is not practically possible to mention/acknowledge all.

AMARNATH MURTHY
amaranth.1960@gmail.com

DGM (E&T)
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