W6. Let D_1 be set of strictly decreasing sequences of positive real numbers with first term equal to 1.For any $\mathbf{x}_{\mathbb{N}} := (x_1, x_2, ..., x_n, ...) \in D_1$ prove that

$$\sum_{n=1}^{\infty} \frac{x_n^3}{x_n + 4x_{n+1}} \geq \frac{4}{9}$$

and find the sequence for which equality occurs.

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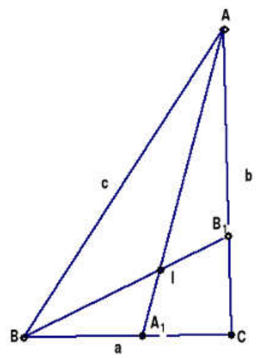
W7. Let $\triangle ABC$ be a right triangle with right angle in C and let be intersection point of bisectors AA_1, BB_1 of acute angles $\angle A$ and $\angle B$, respectively.
Find the right triangle with greatest value of ratio of the "bisectoria" quadrilateral A_1CB_1I area to the triangle $\triangle ABC$ area.



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W8. Let

$$\Delta(x, y, z) = 2xy + 2yz + 2zx - x^2 - y^2 - z^2.$$

Find all triangles with sidelengths a, b, c such that $\Delta(a^n, b^n, c^n) > 0$ for any $n \in \mathbb{N}$.

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W9. Let R, r and s be, respectively,circumradius, inradius and semiperimeter of a triangle.

a) Prove inequality $R^2 - 4r^2 \geq \frac{1}{5} \cdot (s^2 - 27r^2)$;

b) Find the maximum value for constant K such that inequality $R^2 - 4r^2 \geq K(s^2 - 27r^2)$ holds for any triangle;

c) Find the $\lim\limits_{R \to 2r} \frac{R^2 - 4r^2}{s^2 - 27r^2}$.

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