W17. Let $a_1, a_2, ..., a_n, b_1, b_2, ..., b_n$ be positive real numbers such that $a_1 < a_2 < ... < a_n$ and $a_i \in (b_i, b_{i+1})$, $i = 1, 2, ..., n-1$. Let

$$F(x) := \frac{(x-b_1)(x-b_2)...(x-b_n)}{(x-a_1)(x-a_3)...(x-a_n)}.$$

Prove that derivative of function $F(x)$ is negative everywhere where function is defined. (Or, prove that $F'(x) < 0$ for any $x \in Dom(F)$).

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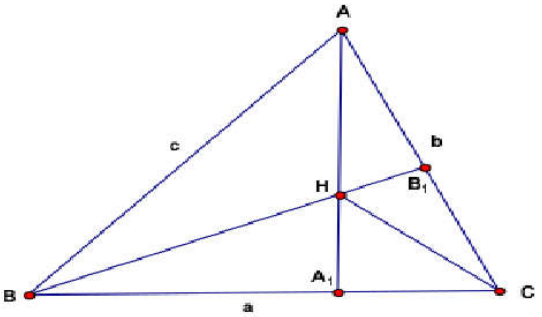
W18. Let F be area of an acute triangle $\triangle ABC$ with bigger angle in C and let AA_1, BB_1, be altitudes from vertices A and B, respectively. Find among such triangles the triangle with greatest value of area of ortho-quadrilateral A_1CB_1H.



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W19. Let $T_n(x)$ be polynomial defined by recurrence $T_{n+1} - 2xT_n - T_{n-1}$, $n \in N$ with initial conditions $T_0 = 1$, $T_1 = x$. (First Kind Chebishev's Polynomials). Prove that

$$\sqrt[n]{T_n(x)} \le 1 + n(x-1), x \ge 1, n \in N$$

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W20. Let p is positive integer such that $p \ge 2$ and let

$$b_n := \sum_{k=0}^{[\log_2 n]} \left[\frac{2^k + n}{2^{k+1}}\right]^p.$$

Find

$$\min_{n \in N} \frac{b_n}{n^p + 2^p - 2}$$

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