<div align="right">Mihály Bencze</div>

W55. If $a \in N^*$, $(a,p) = 1$ where p is a prime, then

$$\prod_{k=1}^{n} \left(a^{p^{k-1}(p-1)} - 1 \right)$$

is divisible by $p^{\frac{n(n+1)}{2}}$.

<div align="right">Mihály Bencze</div>

W56. In all tetrahedron $ABCD$ holds:
1).

$$\frac{1}{h_a h_b} + \frac{2}{h_a h_c} + \frac{1}{h_b h_c} + \frac{2}{h_b h_d} + \frac{1}{h_c h_d} \leq \frac{1}{2r^2}$$

2).

$$\frac{1}{r_a r_b} + \frac{2}{r_a r_c} + \frac{1}{r_b r_c} + \frac{2}{r_b r_d} + \frac{1}{r_c r_d} \leq \frac{2}{r^2}$$

<div align="right">Mihály Bencze</div>

W57. Prove that

$$\int_0^{\frac{\pi}{2}} \prod_{k=1}^{n} \left(\sin^{2k+1} x + \cos^{2k+1} x \right) dx \geq \frac{\pi}{2 \left(\sqrt{2} \right) n^2}; n \in N^*$$

<div align="right">Daniel Sitaru</div>

W58. Let f_n be $n - th$ Fibonacci number defined by recurrence

$$f_{n+1} - f_n - f_{n-1} = 0, n \in N$$

and initial conditions $f_0 = 0, f_1 = 1$.
Prove that $\left(5n^2 + 3n - 2 \right) f_n - 6n f_{n+1}$ is divisible by 50 for any $n \in \mathbb{N}$

<div align="right">Arkady Alt</div>

W59. Let E be a Inner Product Space with dot product $- \cdot -$ and F be proper nonzero subspace. Let $P : \mathbf{E} \longrightarrow \mathbf{E}$ be orthogonal projection $\mathbf{E}$ on $\mathbf{F}$.
a). Prove that for any $\mathbf{x}, \mathbf{y} \in \mathbf{E}$, holds inequality

$$|\mathbf{x} \cdot \mathbf{y} - \mathbf{x} \cdot P(\mathbf{y}) - \mathbf{y} \cdot P(\mathbf{x})| \leq \|\mathbf{x}\| \, \|\mathbf{y}\|$$

b). Determine all cases when equality occurs.

<div align="right">Arkady Alt</div>

W60. Let x, a, h be arbitrary real numbers such that
$x > 0, a \geq -1 , h > 0$ and let sequence (x_n)
defined recursively by $x_1 = x, \ x_{n+1} = \dfrac{n + a}{n + a + h} x_n, \ n \in N \cup \{0\}$.
Explore for which h the infinite sum $\sum\limits_{n=1}^{\infty} x_n$ converges and find it in the case of convergence.

<div align="right">Arkady Alt</div>