If $\widehat{b}=\widehat{0}$ we get from the first equation that $\widehat{a}=\widehat{0}$, so $Y=O_2$.
If $\widehat{b}=-\widehat{2}\widehat{a}=\widehat{p-2}\widehat{a}$, the first equation implies that $\widehat{a}^2\left(\widehat{1}+\frac{p-1}{2}\widehat{p-2}^2\right)=\widehat{0}$
$\Rightarrow \widehat{p-1}\widehat{a}^2=0 \Rightarrow \widehat{a}=\widehat{0} \Rightarrow Y=O_2$.
Thus, the solution of the matrix equation is $X=\widehat{p-1}^{\frac{p-1}{2}}A$. If $p=4i+1$ we have that $\widehat{p-1}^{\frac{p-1}{2}}=\widehat{1}$, so $X=A$ and if $p=4i+3$, then $\widehat{p-1}^{\frac{p-1}{2}}=\widehat{p-1}$, so $X=\widehat{p-1}A=\begin{pmatrix}\widehat{1} & \widehat{p-2}\\ \widehat{1} & \widehat{p-1}\end{pmatrix}$.

Ovidiu Furdui

W13. (Solution by the proposer.) Denoting $\Delta_n := \Delta\left(a^n,b^n,c^n\right)$ we can rewrite original inequality shortly as $\Delta_1\Delta_3 \geq 3\Delta_4$.
Using free parametrization of triangle

$$(a,b,c)=(y+z.z+x,x+y)\,,x,y,z>0,$$

denoting $p:=xy+yz+zx, q:=xyz$ and assuming $x+y+z=1$ due homogeneity of inequality we obtain

$$\Delta_1=4p, \Delta_3=36q-18pq+3q^2-9p^2+4p^3,\ \Delta_4=16\left(1-p\right)\left(4q-(p+q)^2\right)$$

and then

$$h\left(p,q\right):=\frac{\Delta_1\Delta_3-3\Delta_4}{4}=$$

$$=p\left(36q-18pq+3q^2-9p^2+4p^3\right)-12\left(1-p\right)\left(4q-(p+q)^2\right)=$$

$$=3\left(4-3p\right)q^2-6\left(4-7p\right)\left(2-p\right)q+p^2\left(12-21p+4p^2\right)$$

We have

$$p=xy+yz+zx\leq\frac{(x+y+z)^2}{3}=\frac{1}{3}, q=xyz\leq\frac{(x+y+z)^3}{27}=\frac{1}{27}.$$

To prove inequality $h\left(p,q\right)\geq 0$ we need precise range of p and q which give us condition of solvability in real x,y,z of Viet System

$$\begin{cases} x+y+z=1 \\ xy+yz+zx=p \\ xyz=q \end{cases}. \tag{1}$$

Namely, system (1) is solvable in real a, b, c iff it's discriminant

$$D := (x-y)^2 (y-z)^2 (z-x)^2 = p^2 - 4p^3 + 18pq - 4q - 27q^2$$

isn't negative, that is $D \geq 0 \iff$

$$p^2 - 4p^3 + 18pq \; - 4q \; - 27q^2 \geq 0 \iff \begin{cases} 3p \leq 1 \\ \dfrac{4(1-3p)^3}{27} \geq 27\left(q - \dfrac{9p-2}{27}\right)^2 \end{cases},$$

which after rationalizing substitution $p := p(t) = \dfrac{1-t^2}{3}, t \geq 0$ becomes

$$\frac{(1+t)^2(1-2t)}{27} \leq q \leq \frac{(1-t)^2(1+2t)}{27}, \; 0 \leq t \tag{2}$$

Since $q \leq \dfrac{1}{27}$ then

$$\frac{(4-7p)(2-p)}{4-3p} > \frac{1}{27} \iff 27(4-7p)(2-p) > 4-3p$$

and

$$27(4-7p)(2-p) - (4-3p) > 7(1-3p)(20-9p) \geq 0.$$

Therefore, $h(p,q)$ as quadratic function of q is decreasing in

$$q \leq q^* = \frac{(1-t)^2(1+2t)}{27} \leq \frac{1}{27}$$

and then we have

$$h(p,q) = h(p(t), q) \geq h(p(t), q^*) =$$

$$\left(3(4-3p(t))q^{*2} - 6(4-7p(t))(2-p(t))q^* + p^2(t)\left(12 - 21p(t) + 4p^2(t)\right)\right).$$

Since

$$3\left(4-3p\left(t\right)\right)=3\left(3+t^{2}\right),6\left(4-7p\left(t\right)\right)\left(2-p\left(t\right)\right)=\frac{2}{3}\left(t^{2}+5\right)\left(7t^{2}+5\right),$$

$$\left(\frac{1-t^{2}}{3}\right)^{2}\left(12-21\left(\frac{1-t^{2}}{3}\right)+4\left(\frac{1-t^{2}}{3}\right)^{2}\right)=$$

$$=\frac{1}{81}\left(t-1\right)^{2}\left(t+1\right)^{2}\left(4t^{4}+55t^{2}+49\right)$$

then

$$h\left(p\left(t\right),q^{*}\right)=$$

$$=3\left(3+t^{2}\right)\left(\frac{\left(1-t\right)^{2}\left(1+2t\right)}{27}\right)^{2}-\frac{2}{3}\left(t^{2}+5\right)\left(7t^{2}+5\right)\left(\frac{\left(1-t\right)^{2}\left(1+2t\right)}{27}\right)+$$

$$\frac{1}{81}\left(t-1\right)^{2}\left(t+1\right)^{2}\left(4t^{4}+55t^{2}+49\right)=\frac{16}{243}t^{2}\left(t^{2}-2t+4\right)\left(t-1\right)^{4}\geq 0.$$

Arkady Alt

Second solution. We introduce new variables

$$a+b+c=3u,\quad ab+bc+ca=3v^{2},\quad abc=w^{3}$$

$$\Delta(a,b,c)=2(ab+bc+ca)-(a^{2}+b^{2}+c^{2})=4(ab+bc+ca)-(a+b+c)^{2}=12v^{2}-9u^{2}$$

$$\Delta(a^{3},b^{3},c^{3})=4((ab)^{3}+(bc)^{3}+(ca)^{3})-(a^{3}+b^{3}+c^{3})^{2}=$$
$$=4(27v^{6}-27yv^{2}w^{3}+3w^{6})-(27u^{3}-27uv^{2}+3w^{3})^{2}$$

$$\Delta(a^{4},b^{4},c^{4})=4((ab)^{4}+(bc)^{4}+(ca)^{4})-(a^{4}+b^{4}+c^{4})^{2}=$$
$$=4((9v^{4}-6uw^{3})^{2}-2w^{6}(9u^{2}-6v^{2}))-(81u^{4}-108u^{2}v^{2}+18v^{4}+12uw^{3})^{2}$$