is an integer number.

Dorin Mărghidanu

W28. For any fixed natural n let
$D_n := \{(x,y)\,|x,y \in Z \text{ and } F_{n+1}x - F_n y = 1\}$, where F_n is n^{th} Fibonacci number. Find:

a). $\min\limits_{(x,y)\in D_n} |x+y|$; b). $\min\limits_{(x,y)\in D_n} (|x|+|y|)$; c). $\min\limits_{(x,y)\in D_n} \left(x^2+y^2\right)$

Arkady Alt

W29. Let a,b,c be side lengths of a triangle ABC and x,y,z be non-negative real numbers such that $x+y+z=1$ and let R be circumradius of this triangle. Prove that

$$a^2yz + b^2zx + c^2xy \le R^2$$

Arkady Alt

W30. Let $P_m(x) := \sum\limits_{k=0}^{m} \frac{(-1)^k x^{m-k}}{k+1}, m \in N$. For any $m \in N$ calculate:

$$\lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)^{n^m}}{e^{P_{m-1}(n)}}$$

Arkady Alt

W31. Find $\inf\limits_{(x,y)\in D} (x-1)(y-1)$ where

$$D := \{(x,y)\,|x,y \in R_+, x \ne y \text{ and } x^y = y^x\}$$

Arkady Alt

W32. Calculate

$$\sum_{n=1}^{\infty} H_n \left[\zeta^2(2) - \left(1 + \frac{1}{2^2} + \cdots + \frac{1}{n^2}\right)^2 - \frac{2\zeta(2)}{n}\right],$$

where $H_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$ denotes the n^{th} harmonic number.

Ovidiu Furdui and Alina Sîntămărian