Undergraduate problems

U331. Find all positive integers $a > b \geq 2$ such that

$$a^b - a = b^a - b.$$

Proposed by Mircea Becheanu, Bucharest, Romania

U332. Find $\inf_{(x,y)\in D} (x+1)(y+1)$, where $D = \{(x,y) \mid x, y \in \mathbb{R}_+, x \neq y, \text{and } x^y = y^x\}$.

Proposed by Arkady Alt, San Jose, USA

U333. Evaluate

$$\prod_{n\geq 0}\left(1 - \frac{2^{2^n}}{2^{2^{n+1}}+1}\right).$$

Proposed by Titu Andreescu, University of Texas at Dallas, USA

U334. Prove that if $x \in \mathbb{R}$ with $|x| \geq e$ then, $e^{|x|} > \left(\frac{e^2+x^2}{2e}\right)^e$, while the inequality is reversed if $|x| \leq e$.

Proposed by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain

U335. Let p, $a_1, \dots, a_n$, $b_1, \dots, b_n$ be positive real numbers. Prove that

$$a_1\left(\frac{a_1}{b_1}\right)^p + \cdots + a_n\left(\frac{a_1+\cdots+a_n}{b_1+\cdots+b_n}\right)^p < \left(\frac{p+1}{p}\right)^p\left(\frac{a_1^{p+1}}{b_1^p} + \cdots + \frac{a_n^{p+1}}{b_n^p}\right).$$

Proposed by Nairi Sedrakyan, Armenia

U336. Find a closed form for the sum $E_n = \sum_{k=0}^{n}\binom{n}{2k+1}3^k$. Deduce the value of constant c such that $0 < \lim_{n\to\infty}\frac{E_n}{c^n} < \infty$, and the limit itself.

Proposed by Ángel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.