Undergraduate problems

U181. Consider sequences $(a_n)_{n\geq 0}$ and $(b_n)_{n\geq 0}$, where $a_0 = b_0 = 1$, $a_{n+1} = a_n + b_n$, and $b_{n+1} = (n^2+n+1)a_n + b_n,\ n \geq 1$. Evaluate $\lim_{n\to\infty} B_n$, where

$$B_n = \frac{(n+1)^2}{\sqrt[n+1]{a_{n+1}}} - \frac{n^2}{\sqrt[n]{a_n}}.$$

Proposed by Neculai Stanciu, George Emil Palade, Buzau, Romania

U182. Find all continuous functions f on $[0,1]$ such that $f(x) = c$ if $x \in \left[0, \frac{1}{2}\right]$ and $f(x) = f(2x-1)$ if $x \in \left(\frac{1}{2}, 1\right]$, where c is a given constant.

Proposed by Arkady Alt, San Jose, California, USA

U183. Let m and n be positive integers. Prove that

$$\sum_{k=0}^{n} \frac{1}{k+m+1}\binom{n}{k} \leq \frac{(m+2n)^{m+n+1} - n^{m+n+1}}{(m+n+1)(m+n)^{m+n+1}}.$$

Proposed by Dorin Andrica, Babes-Bolyai University, Cluj-Napoca, Romania

U184. Let $f, g : [a,b] \to \mathbb{R}$ be differentiable functions such that $\int_a^b f(x)dx = 0$. Prove that there is some $c \in (a,b)$ satisfying

$$f'(c)\int_c^b g(x)dx + g'(c)\int_c^b f(x)dx = 2f(c)g(c).$$

Proposed by Duong Viet Thong, National Economics University, Vietnam

U185. Determine if there is a non-constant complex analytic function satisfying the conditions:

(i) $f(f(z)) = f(z)$ for all complex numbers z

(ii) there is a complex number z_0, such that $f(z_0) \neq z_0$.

Proposed by Harun Immanuel, Airlangga University, Indonesia