## Senior problems

S229. Let $a$, $b$, $c$ be the side-lengths of a triangle and let $R$ be its circumradius. Prove that

$$a^3 + b^3 + c^3 \leq 16R^3.$$

*Proposed by Arkady Alt, San Jose, USA and Ivan Borsenco, MIT, USA*

S230. Let $x$, $y$, $z$ be positive real numbers such that

$$xy + yz + zx \geq \frac{1}{\sqrt{x^2 + y^2 + z^2}}.$$

Prove that $x + y + z \geq \sqrt{3}$.

*Proposed by Titu Andreescu, University of Texas at Dallas, USA*

S231. Let $ABC$ be a triangle with circumcenter $O$. Let $X$, $Y$, $Z$ be the circumcenters of triangles $BCO$, $CAO$, $ABO$ respectively. Furthermore, let $K$ be the circumcenter of triangle $XYZ$. Prove that $K$ lies on the Euler line of triangle $ABC$.

*Proposed by Andrew Kirk, Mearns Castle High School, UK*

S232. Let $x$, $y$, $z$ be real numbers such that $x + y + z = 0$ and $xy + yz + zx = -3$. Determine the extreme values of $x^4y + y^4z + z^4x$.

*Proposed by Marius Stanean, Zalau, Romania*

S233. In triangle $ABC$ with $\angle C = 60^\circ$, let $AA'$ and $BB'$ be the angle bisectors of $\angle A$ and $\angle B$. Prove that

$$\frac{a+b}{A'B'} \leq \left(1 + \frac{c}{a}\right)\left(1 + \frac{c}{b}\right).$$

*Proposed by Titu Andreescu, University of Texas at Dallas, USA*

S234. Let $ABC$ be a triangle. Denote by $D$, $E$, $F$ the feet of the internal angle bisectors such that $D \in (BC)$, $E \in (CA)$, $F \in (AB)$ and by $(I_a, r_a), (I_b, r_b), (I_c, r_c)$ its three excircles. If $\tau$ denotes the Feuerbach point of triangle $ABC$, prove that there is a choice of signs $+$ and $-$ such that the following equality holds

$$\pm D\tau \cdot \frac{I_aI}{I_aD} \cdot \sqrt{R + 2r_a} \pm E\tau \cdot \frac{I_bI}{I_bE} \cdot \sqrt{R + 2r_b} \pm F\tau \cdot \frac{I_cI}{I_cF} \cdot \sqrt{R + 2r_c} = 0.$$

*Proposed by Cosmin Pohoata, Princeton University, USA*