Senior problems

S199. In triangle ABC let BB' and CC' be the angle bisectors of $\angle B$ and $\angle C$. Prove that

$$B'C' \geq \frac{2bc}{(a+b)(a+c)}\left[(a+b+c)\sin\frac{A}{2} - \frac{a}{2}\right].$$

Proposed by Titu Andreescu, University of Texas at Dallas, USA

S200. On each vertex of the regular hexagon $A_1A_2A_3A_4A_5A_6$ we place a rod. On each rod we have a_i rings, where a_i corresponds to the vertex A_i. Taking a ring from any three adjacent rods we can create chains of three rings. What is the maximum number of such chains that we can create?

Proposed by Arkady Alt, San Jose, USA

S201. Prove that in any triangle,

$$r_a \leq 4R\sin^3\left(\frac{A}{3} + \frac{\pi}{6}\right).$$

Proposed by Dorin Andrica, Babes-Bolyai University, Romania

S202. Let a and b be integers such that $a^2m - b^2n = a - b$, for some consecutive integers m and n. Prove that $\gcd(a,b) = \sqrt{|a-b|}$.

Proposed by Titu Andreescu, University of Texas at Dallas, USA

S203. Let ABC be a triangle, and P a point not lying on its sides. Call XYZ the cevian triangle of P with respect ABC and consider the points Y_a, Z_a of intersection of BC with the parallel lines to AX through Y and Z respectively. Prove that AX, YZ_a, Y_aZ concur in a point Q that satisfies the cross-ratio

$$(AXPQ) = \frac{AP}{AX}.$$

Proposed by Francisco Javier Garcia Capitan, Spain

S204. Find all positive integers k and n such that $k^n - 1$ and n are divisible by precisely the same primes.

Proposed by Tigran Hakobyan, Yerevan, Armenia