Olympiad problems

O295. Let ABC be a triangle with circumcircle Γ and incircle ω. Let D, E, F be the tangency points of ω with BC, CA, AB, respectively, let Q be the second intersection of AD with Γ, and let the T be the intersection of the tangents at B and C with respect to Γ. Furthermore, let QT intersect Γ for the second time at R. Prove that AR, EF, BC are concurrent.

Proposed by Faraz Masroor, Gulliver Preparatory, Florida, USA

O296. Let a, b, c, x, y, z be positive real numbers such that $x+y+z=1$ and

$$2ab+2bc+2ca > a^2+b^2+c^2.$$

Prove that

$$a\,(x+3yz)+b(y+3xz)+c\,(z+3xy) \le \frac{2}{3}(a+b+c).$$

Proposed by Arkady Alt, San Jose, California, USA

O297. Let m be a positive integer. Prove that $\phi(n)$ divides mn, only for finitely many square-free integers n, where ϕ is Euler's totient function.

Proposed by Ivan Borsenco, Massachusetts Institute of Technology, USA

O298. Cells of an 11×11 square are colored in n colors. It is known that the number of cells of each color is greater than 6 and less than 14. Prove that one can find a row and a column whose cells are colored in at least four different colors.

Proposed by Nairi Sedrakyan, Yerevan, Armenia

O299. Let n be a square-free positive integer. Find the number of functions $f:\{1,2,\ldots,n\}\to\{1,2,\ldots,n\}$ such that $f(1)f(2)\cdots f(n)$ divides n.

Proposed by Mihai Piticari, Campulung Moldovenesc, Romania

O300. Let a,b,c be positive real numbers such that $a^2+b^2+c^2+abc=4$. Prove that

$$\sqrt{1-abc}(3-a-b-c) \ge |(a-1)(b-1)(c-1)|.$$

Proposed by Marius Stanean, Zalau, Romania