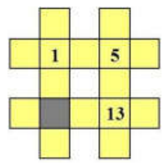
Junior problems

J91. The squares in the figure below are labeled 1 through 16 such that the sum of the numbers in each row and each column is the same. The positions of $1, 5$, and 13 are given.



Prove that there is only one possibility for the number in the darkened square and find this number.

Proposed by Ivan Borsenco, Massachusetts Institute of Technology, USA

J92. Find all primes $q_1, q_2, \ldots, q_6$ such that $q_1^2 = q_2^2 + \cdots + q_6^2$.

Proposed by Titu Andreescu, University of Texas at Dallas, USA

J93. Let a and b be positive real numbers. Prove that

$$\frac{a^6+b^6}{a^4+b^4} \geq \frac{a^4+b^4}{a^3+b^3} \cdot \frac{a^2+b^2}{a+b}.$$

Proposed by Arkady Alt, San Jose, California, USA

J94. Prove that the equation $x^3 + y^3 + z^3 + w^3 = 2008$ has infinitely many solutions in integers.

Proposed by Titu Andreescu, University of Texas at Dallas, USA

J95. Let ABC be a triangle and let I_a, I_b, I_c be its excenters. Denote by O_a, O_b, O_c the circumcenters of triangles I_aBC, I_bAC, I_cAB. Prove that the area of triangle $I_aI_bI_c$ is twice the area of hexagon $O_aCO_bAO_cB$.

Proposed by Mehmet Sahin, Ankara, Turkey

J96. Let n be an integer. Find all integers m such that $a^m + b^m \geq a^n + b^n$ for all positive real numbers a and b with $a + b = 2$.

Proposed by Oleg Mushkarov, Bulgarian Academy of Sciences, Sofia, Bulgaria