Junior problems

J85. Let a and b be positive real numbers. Prove that

$$\sqrt[3]{\frac{(a+b)(a^2+b^2)}{4}} \geq \sqrt{\frac{a^2+ab+b^2}{3}}.$$

Proposed by Arkady Alt, San Jose, California, USA

J86. A triangle is called α-angular if none of its angles exceeds α degrees. Find the least α for which each non α-angular triangle can be dissected into some α-angular triangles.

Proposed by Titu Andreescu, University of Texas at Dallas and Gregory Galperin, Eastern Illinois University, USA

J87. Prove that for any acute triangle ABC, the following inequality holds:

$$\frac{1}{-a^2+b^2+c^2} + \frac{1}{a^2-b^2+c^2} + \frac{1}{a^2+b^2-c^2} \geq \frac{1}{2Rr}.$$

Proposed by Mircea Becheanu, Bucharest, Romania

J88. Find the greatest n for which there are points $P_1, P_2, \ldots, P_n$ in the plane such that each triangle whose vertices are among $P_1, P_2, \ldots P_n$, has a side less than 1 and a side greater than 1.

Proposed by Ivan Borsenco, University of Texas at Dallas, USA

J89. Let A and B lie on circle $\mathcal{C}$ of center O and let C be the point on the small arc AB such that OA is the external angle bisector of $\angle BOC$. Denote by M the midpoint of BC and by N the intersection of AM and OC. Prove that the intersection of the angle bisector of $\angle BOC$ with the circle of center O and radius ON is the center of the circle tangent to lines OB and OC, and also internally tangent to $\mathcal{C}$.

Proposed by Francisco Javier Garcia Capitan, Spain

J90. For a fixed positive integer n let $a_k = 2^{2^{k-n}} + k$, $k = 0, 1, \ldots, n$. Prove that

$$(a_1 - a_0) \cdots (a_n - a_{n-1}) = \frac{7}{a_1 + a_0}.$$

Proposed by Titu Andreescu, University of Texas at Dallas