Junior problems

J289. Let a be a real number such that $0 \le a < 1$. Prove that

$$\left\lfloor a\left(1+\left\lfloor\frac{1}{1-a}\right\rfloor\right)\right\rfloor+1=\left\lfloor\frac{1}{1-a}\right\rfloor.$$

Proposed by Arkady Alt, San Jose, California, USA

J290. Let a, b, c be nonnegative real numbers such that $a+b+c=1$. Prove that

$$\sqrt[3]{13a^3+14b^3}+\sqrt[3]{13b^3+14c^3}+\sqrt[3]{13c^3+14a^3}\ge 3.$$

Proposed by Titu Andreescu, University of Texas at Dallas, USA

J291. Let ABC be a triangle such that $\angle BCA = 2\angle ABC$ and let P be a point in its interior such that $PA = AC$ and $PB = PC$. Evaluate the ratio of areas of triangles PAB and PAC.

Proposed by Panagiote Ligouras, Noci, Italy

J292. Find the least real number k such that for every positive real numbers x, y, z, the following inequality holds:

$$\prod_{cyc}(2xy+yz+zx)\le k(x+y+z)^6.$$

Proposed by Dorin Andrica, Babes-Bolyai University, Romania

J293. Find all positive integers x, y, z such that

$$(x+y^2+z^2)^2-8xyz=1.$$

Proposed by Aaron Doman, University of California, Berkeley , USA

J294. Let a, b, c be nonnegative real numbers such that $a+b+c=3$. Prove that

$$1\le(a^2-a+1)(b^2-b+1)(c^2-c+1)\le 7.$$

Proposed by An Zhen-ping, Xianyang Normal University, China