Junior problems

J259. Among all triples of real numbers (x, y, z) which lie on a unit sphere $x^2 + y^2 + z^2 = 1$ find a triple which maximizes $\min(|x-y|, |y-z|, |z-x|)$.

Proposed by Arkady Alt, San Jose, California, USA

Solution by Polyahedra, Polk State College, USA

Without loss of generality, assume that $x \le y \le z$. Then

$$m = \min(|x-y|, |y-z|, |z-x|) = \min(y-x, z-y) \le \frac{z-x}{2}.$$

Now $x^2 + y^2 + z^2 = 1$ can be written as $(z-x)^2 = 2 - (z+x)^2 - 2y^2$. Thus $(z-x)^2 \le 2$, with equality if and only if $z + x = 0 = y$. Hence $m \le \frac{\sqrt{2}}{2}$, with the maximum of $\frac{\sqrt{2}}{2}$ attained at $(x, y, z) = \left(\frac{-\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)$.

Also solved by Antonio Trusiani, Università di Roma "Tor Vergata", Roma, Italy; Daniel Lasaosa, Universidad Pública de Navarra, Spain; Alessandro Ventullo, Milan, Italy; Radouan Boukharfane, Polytechnique de Montreal, Canada.