Junior problems

J259. Among all triples of real numbers (x, y, z) which lie on a unit sphere $x^2+y^2+z^2=1$ find a triple which maximizes $\min(|x-y|, |y-z|, |z-x|)$.

Proposed by Arkady Alt , San Jose, California, USA

J260. Solve in integers the equation

$$x^4 - y^3 = 111.$$

Proposed by José Hernández Santiago, Oaxaca, México

J261. Let $A_1 \dots A_n$ be a polygon inscribed in a circle with center O and radius R. Find the locus of points M on the circumference such that

$$A_1M^2 + \cdots + A_nM^2 = 2nR^2.$$

Proposed by Ivan Borsenco, Massachusetts Institute of Technology, USA

J262. Find all positive integers m, n such that $\binom{m+1}{n} = \binom{n}{m+1}$.

Proposed by Roberto Bosch Cabrera, Havana, Cuba.

J263. The n-th pentagonal number is given by the formula $p_n = \frac{n(3n-1)}{2}$. Prove that there are infinitely many pentagonal numbers that can be written as a sum of two perfect squares of positive integers.

Proposed by José Hernández Santiago, Oaxaca, México

J264. In triangle ABC, $2\angle A = 3\angle B$. Prove that

$$(a^2 - b^2)(a^2 + ac - b^2) = b^2c^2.$$

Proposed by Titu Andreescu, University of Texas at Dallas, USA