Junior problems

J193. Let $ABCD$ be a square of center O. The parallel through O to AD intersects AB and CD at M and N and a parallel to AB intersects diagonal AC at P. Prove that

$$OP^4 + \left(\frac{MN}{2}\right)^4 = MP^2 \cdot NP^2.$$

Proposed by Titu Andreescu, University of Texas at Dallas, USA

J194. Let a, b, c be the side-lenghts of a triangle with the largest side c. Prove that

$$\frac{ab\,(2c+a+b)}{(a+c)\,(b+c)} \le \frac{a+b+c}{3}.$$

Proposed by Arkady Alt, San Jose, California, USA

J195. Find all primes p and q such that both $pq - 555p$ and $pq + 555q$ are perfect squares.

Proposed by Titu Andreescu, University of Texas at Dallas, USA

J196. Let I be the incenter of triangle ABC and let A', B', C' be the feet of altitudes from vertices A, B, C. If $IA' = IB' = IC'$, then prove that triangle ABC is equilateral.

Proposed by Dorin Andrica and Liana Topan, Babes-Bolyai University, Romania

J197. Let x, y, z be positive real numbers. Prove that

$$\sqrt{2\left(x^2y^2 + y^2z^2 + z^2x^2\right)\left(\frac{1}{x^3} + \frac{1}{y^3} + \frac{1}{z^3}\right)} \ge x\sqrt{\frac{1}{y} + \frac{1}{z}} + y\sqrt{\frac{1}{z} + \frac{1}{x}} + z\sqrt{\frac{1}{x} + \frac{1}{y}}.$$

Proposed by Vazgen Mikayelyan, Yerevan, Armenia

J198. Find all pairs (x, y) for which $x! + y! + 3$ is a perfect cube.

Proposed by Titu Andreescu, University of Texas at Dallas, USA