PROBLEMS

Readers are invited to submit solutions, comments and generalizations to any problem in this section. Moreover, readers are encouraged to submit problem proposals. Please see submission guidelines inside the back cover or online.

To facilitate their consideration, solutions should be received by the editor by **August 1, 2016**, *although late solutions will also be considered until a solution is published.*

The editor thanks André Ladouceur, Ottawa, ON, for translations of the problems.

4041. *Proposed by Arkady Alt.*

Let a, b and c be the side lengths of a triangle ABC. Let AA', BB' and CC' be the heights of the triangle and let $a_p = B'C', b_p = C'A'$ and $c_p = A'B'$ be the sides of the orthic triangle. Prove that:

a) $a^2(b_p + c_p) + b^2(c_p + a_p) + c^2(a_p + b_p) = 3abc$;

b) $a_p + b_p + c_p \leq s$, where s is the semiperimeter of ABC.

4042. *Proposed by Leonard Giugiuc and Diana Trailescu.*

Let a, b and c be real numbers in $[0, \pi/2]$ such that $a + b + c = \pi$. Prove the inequality

$$2\sqrt{2}\sin\frac{a}{2}\sin\frac{b}{2}\sin\frac{c}{2} \geq \sqrt{\cos a \cos b \cos c}.$$

4043. *Proposed by Michel Bataille.*

Suppose that the lines m and n intersect at A and are not perpendicular. Let B be a point on n, with $B \neq A$. If F is a point of m, distinct from A, show that there exists a unique conic $\mathcal{C}_F$ with focus F and focal axis BF, intersecting n orthogonally at A. Given $\epsilon > 0$, how many of the conics $\mathcal{C}_F$ have eccentricity ϵ?

4044. *Proposed by Dragoljub Milošević.*

Let x, y, z be positive real numbers such that $x + y + z = 1$. Prove that

$$\frac{x+1}{x^3+1} + \frac{y+1}{y^3+1} + \frac{z+1}{z^3+1} \leq \frac{27}{7}.$$

4045. *Proposed by Galav Kapoor.*

Suppose that we have a natural number n such that $n \geq 10$. Show that by changing at most one digit of n, we can compose a number of the form $x^2 + y^2 + 10z^2$, where x, y, z are integers.