4031. *Proposed by D. M. Bătineţu-Giurgiu and Neculai Stanciu.*

Prove that

$$\frac{2F_1^4+F_2^4+F_3^4}{F_1^2+F_3^2}+\frac{2F_2^4+F_3^4+F_4^4}{F_2^2+F_4^2}+\cdots+\frac{2F_n^4+F_1^4+F_2^4}{F_n^2+F_2^2}>2F_nF_{n+1},$$

where F_n represents the nth Fibonacci number ($F_0=0, F_1=1$ and $F_{n+2}=F_n+F_{n+1}$ for all $n\geq 0$).

4032. *Proposed by Dan Stefan Marinescu and Leonard Giugiuc.*

Prove that in any triangle ABC with sides a,b and c, inradius r and exradii r_a,r_b,r_c, we have:

$$\sqrt{ab}+\sqrt{bc}+\sqrt{ca}\geq 2\sqrt{3r(r_a+r_b+r_c)}.$$

4033. *Proposed by Salem Malikic.*

Let $\alpha_1,\ldots,\alpha_n,\beta_1,\ldots,\beta_n$ be positive real numbers and $x_1,\ldots,x_n$ be real numbers such that $x_1+\cdots+x_n=1$ and $\alpha_ix_i+\beta_i\geq 0$ for all $i=1,\ldots,n$. Find the maximum value of

$$\sqrt{\alpha_1x_1+\beta_1}+\sqrt{\alpha_2x_2+\beta_2}+\cdots+\sqrt{\alpha_nx_n+\beta_n}.$$

4034. *Proposed by Michel Bataille.*

Evaluate

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{16^n}\sum_{k=0}^{2n}\frac{(-1)^k}{2n+1-k}\binom{2k}{k}\binom{4n-2k}{2n-k}.$$

4035. *Proposed by Daniel Sitaru and Leonard Giugiuc.*

Let a and b be two real numbers such that $ab=225$. Find all real solutions (in real 2×2 matrices) to the matrix equation

$$X^3-5X^2+6X=\begin{pmatrix}15 & a\\ b & 15\end{pmatrix}.$$

4036. *Proposed by Arkady Alt.*

Let a,b and c be non-negative real numbers. Prove that for any real $k\geq\frac{11}{24}$ we have:

$$k(ab+bc+ca)(a+b+c)-(a^2c+b^2a+c^2b)\leq\frac{(3k-1)(a+b+c)^3}{9}.$$