PROBLEMS

Readers are invited to submit solutions, comments and generalizations to any problem in this section. Moreover, readers are encouraged to submit problem proposals. Please see submission guidelines inside the back cover or online.

To facilitate their consideration, solutions should be received by the editor by **May 1, 2016**, *although late solutions will also be considered until a solution is published.*

The editor thanks André Ladouceur, Ottawa, ON, for translations of the problems.

An asterisk ($\star$) *after a number indicates that a problem was proposed without a solution.*

4021. *Proposed by Arkady Alt.*

Let $(\overline{\mathbf{a}}_n)_{n\geq 0}$ be a sequence of Fibonacci vectors defined recursively by $\overline{\mathbf{a}}_0 = \overline{\mathbf{a}}, \overline{\mathbf{a}}_1 = \overline{\mathbf{b}}$ and $\overline{\mathbf{a}}_{n+1} = \overline{\mathbf{a}}_n + \overline{\mathbf{a}}_{n-1}$ for all integers $n \geq 1$. Prove that, for all integers $n \geq 1$, the sum of vectors $\overline{\mathbf{a}}_0 + \overline{\mathbf{a}}_1 + \cdots + \overline{\mathbf{a}}_{4n+1}$ equals $k\overline{\mathbf{a}}_i$ for some i and constant k.

4022. *Proposed by Leonard Giugiuc.*

In a triangle ABC, let internal angle bisectors from angles A, B and C intersect the sides BC, CA and AB in points D, E and F and let the incircle of ΔABC touch the sides in M, N, and P, respectively. Show that

$$\frac{PA}{PB} + \frac{MB}{MC} + \frac{NC}{NA} \geq \frac{FA}{FB} + \frac{DB}{DC} + \frac{EC}{EA}.$$

4023. *Proposed by Ali Behrouz.*

Find all functions $f : \mathbb{R}^+ \mapsto \mathbb{R}^+$ such that for all $x, y \in \mathbb{R}$ with $x > y$, we have

$$f\left(\frac{x}{x-y}\right) + f(xf(y)) = f(xf(x)).$$

4024. *Proposed by Leonard Giugiuc.*

Let a, b, c and d be real numbers such that $a^2 + b^2 + c^2 + d^2 = 4$. Prove that

$$abc + abd + acd + bcd + 4 \geq a + b + c + d$$

and determine when equality holds.

4025. *Proposed by Dragoljub Miloşević.*

Prove that for positive numbers a, b and c, we have

$$\sqrt[3]{\left(\frac{a}{2b+c}\right)^2} + \sqrt[3]{\left(\frac{b}{2c+a}\right)^2} + \sqrt[3]{\left(\frac{c}{2a+b}\right)^2} \geq \sqrt[3]{3}.$$