3945. *Proposed by J. Chris Fisher.*

Given circles (A) and (B) with centres A and B, and a circle (C) with centre C that meets (A) in points A_1 and A_2 that are not on (B), and meets (B) in points B_1 and B_2 that are not on (A), prove that the unique conic with foci A and B that is tangent to the perpendicular bisector of A_2B_2 is tangent also to the perpendicular bisector ℓ of A_1B_1.

3946. *Proposed by George Apostolopoulos.*

Prove that in any triangle ABC

a) $\dfrac{a^2}{w_bw_c}+\dfrac{b^2}{w_aw_c}+\dfrac{c^2}{w_aw_b}\geq 4,$

b) $\left(\dfrac{a}{w_bw_c}\right)^2+\left(\dfrac{b}{w_aw_c}\right)^2+\left(\dfrac{c}{w_aw_b}\right)^2\geq\left(\dfrac{4}{3R}\right)^2,$

where R is the circumradius of ABC and w_a, w_b, w_c are the lengths of the internal bisectors of the angle opposite of the sides of lengths a, b, c, respectively.

3947. *Proposed by Michel Bataille.*

Let $A_1A_2A_3$ be a non-isosceles triangle and I its incenter. For $i = 1, 2, 3$, let D_i be the projection of I onto $A_{i+1}A_{i+2}$ and U_i, V_i be the respective projections of A_{i+1}, A_{i+2} onto the line IA_i (indices are taken modulo 3). Prove that

(a) $\dfrac{U_1D_1}{V_1D_1}\cdot\dfrac{U_2D_2}{V_2D_2}\cdot\dfrac{U_3D_3}{V_3D_3}=\dfrac{U_1D_2}{V_1D_3}\cdot\dfrac{U_2D_3}{V_2D_1}\cdot\dfrac{U_3D_1}{V_3D_2}=1,$

(b) $\dfrac{[D_1U_1V_1]}{\sin^2\frac{\alpha_2-\alpha_3}{2}}+\dfrac{[D_2U_2V_2]}{\sin^2\frac{\alpha_3-\alpha_1}{2}}+\dfrac{[D_3U_3V_3]}{\sin^2\frac{\alpha_1-\alpha_2}{2}}=[A_1A_2A_3]$, where α_i is the angle of $\Delta A_1A_2A_3$ at vertex A_i $(i = 1, 2, 3)$ and $[XYZ]$ denotes the area of ΔXYZ.

3948. *Proposed by George Apostolopoulos.*

Let $a_1, a_2, \ldots a_n$ be real numbers such that $a_1 > a_2 > \ldots > a_n$. Prove that

$$\frac{1}{a_1-a_2}+\frac{1}{a_2-a_3}+\cdots+\frac{1}{a_{n-1}-a_n}+a_1-a_n\geq 2(n-1).$$

When does the equality hold ?

3949. *Proposed by Arkady Alt.*

For any positive real a and b, find

$$\lim_{n\to\infty}\left((n+1)\left(\frac{a^{\frac{1}{n+1}}+b^{\frac{1}{n+1}}}{2}\right)^{n+1}-n\left(\frac{a^{\frac{1}{n}}+b^{\frac{1}{n}}}{2}\right)^{n}\right).$$