and the number of students obtaining an A in Test 2 is

$$\frac{1}{3}(3x+0) = x.$$

Finally, the number of students obtaining at least one A is

$$\frac{1}{2}(x+3x+0) = 2x.$$

Therefore, the number of students obtaining an A on both tests is

$$x + x - 2x = 0,$$

as desired.

3932. *Proposed by Arkady Alt.*

Let x and y be natural numbers satisfying equation $x^2 - 14xy + y^2 - 4x = 0$. Find $\gcd(x, y)$ in terms of x and y.

We received eight correct submissions and one incomplete solution. We present the solution by Oliver Geupel and a remark by Dionne Bailey, Elsie Campbell, and Charles Diminnie.

We show that x is a perfect square and $\gcd(x, y) = 2\sqrt{x}$. Rewriting the given equation as

$$\left(y - 7x - 2\sqrt{x(12x+1)}\right)\left(y - 7x + 2\sqrt{x(12x+1)}\right) = 0,$$

we obtain

$$y = 7x \pm 2\sqrt{x(12x+1)}.$$

Therefore, the product of the co-prime numbers x and $12x+1$ is a perfect square, which means that x and $12x+1$ are perfect squares, $x = z^2$ and $12x+1 = u^2$ with relatively prime positive integers z and u. If the number x were odd then

$$u^2 = 12x + 1 \equiv 5 \pmod{8},$$

which is impossible. Thus, x is even and therefore z is even. Consequently,

$$\gcd(x, y) = \gcd\left(x, 2\sqrt{x(12x+1)}\right) = \gcd(z^2, 2zu) = 2z.$$

Hence the result.

Remark, by Dionne Bailey, Elsie Campbell, and Charles Diminnie.

By rewriting the original equation in the form

$$(24x+1)^2 - 12(y-7x)^2 = 1$$

and considering solutions of the resulting Pell's Equation

$$u^2 - 12v^2 = 1$$

(with $u = 24x + 1$ and $v = |y - 7x|$), we can generate solutions of our equation. Some of the smaller solutions are listed in the following table. Note that $\gcd(x, y) = 2\sqrt{x}$ in each case.

x	y	d
4	56	4
784	56	56
784	10,920	56
152,100	10,920	780
152,100	2,118,480	780
29,506,624	2,118,480	10,864

3933. *Proposed by Dragoljub Milošević.*

Let $ABCDEFG$ be a regular heptagon. Prove that

$$\frac{AD^3}{AB^3} - \frac{AB + 2AC}{AD - AC} = 1.$$

Thirteen correct solutions were received. We present four solutions after some preliminaries and editor comments.

Preliminaries. Let $ABCDEFG$ be a regular heptagon having sides of length a, short diagonals (*e.g.* AC) of length b and long diagonals (*e.g.* AD) of length c. Let $\theta = \pi/7$, so that $a = 2R\sin\theta$, $b = 2R\sin 2\theta$ and $c = 2R\sin 3\theta$, where R is the circumradius.

Five solvers based their solution of the use of some of the relationships

$$a^2 + ac = b^2;$$

$$b^2 + ab = c^2;$$

$$a^2 + bc = c^2; \qquad (1)$$

$$ac + ab = bc. \qquad (2)$$

These can be verified by applying Ptolemy's theorem to the respective cyclic quadrilaterals $ABCD$, $ACEG$, $ADEG$, $ADFG$. However, one solver used the trigonometic representations for a, b and c to obtain (1) and (2).

Four solvers used trigonometry. The result is equivalent to

$$\sin^3 3\theta(\sin 3\theta - \sin 2\theta) = \sin^3\theta(\sin\theta + \sin 2\theta + \sin 3\theta). \qquad (3)$$

Three of these followed the strategy of Solution 4.