

**3843.** *Proposed by George Apostolopoulos, Messolonghi, Greece.*

Let  $a, b$  be distinct real numbers such that

$$a^4 + b^4 - 3(a^2 + b^2) + 8 \leq 2(a + b)(2 - ab).$$

Find the value of the expression

$$A = (ab)^n + (ab + 1)^n + (ab + 2)^n,$$

where  $n$  is a positive integer.

**3844.** *Proposed by Michel Bataille, Rouen, France.*

Find the intersection of the surface with equation

$$(x^2 + y^2)^2 + (y^2 + z^2)^2 + (z^2 + x^2)^2 = (x + y)(y + z)(z + x)$$

with the plane  $x + y + z = 2$ .

**3845.** *Proposed by Dao Thanh Oai, Kien Xuong, Thai Binh, Viet Nam.*

Let the six points  $A_1, A_2, \dots, A_6$  lie in that order on a circle, and the six points  $B_1, B_2, \dots, B_6$  lie in that order on another circle. If the quadruples  $A_i, A_{i+1}, B_{i+1}, B_i$  lie on circles with centres  $C_i$  for  $i = 1, 2, \dots, 5$ , then prove that  $A_6, A_1, B_1, B_6$  must also lie on a circle. Furthermore, if  $C_6$  is the centre of the new circle, then prove that lines  $C_1C_4, C_2C_5$ , and  $C_3C_6$  are concurrent.

**3846.** *Proposed by Arkady Alt, San Jose, CA, USA.*

Let  $r$  be a positive real number. Prove that the inequality

$$\frac{1}{1+a+a^2} + \frac{1}{1+b+b^2} + \frac{1}{1+c+c^2} \geq \frac{3}{1+r+r^2}$$

holds for any positive  $a, b, c$  such that  $abc = r^3$  if and only if  $r \geq 1$ .

**3847.** *Proposed by Jung In Lee, Seoul Science High School, Seoul, Republic of Korea.*

Prove that there are no distinct positive integers  $a, b, c$  and nonnegative integer  $k$  that satisfy the conditions

$$a^{b+k} \mid b^{a+k}, \quad b^{c+k} \mid c^{b+k}, \quad c^{a+k} \mid a^{c+k}.$$

**3848.** *Proposed by Rudolf Fritsch, University of Munich, Munich, Germany.*

We define an altitude of the plane  $(2n + 1)$ -gon  $A_0A_1 \dots A_{2n}$  to be the line through vertex  $A_i$  perpendicular to the *opposite side*  $A_{i-n}A_{i+n}$  (where indices are reduced modulo  $2n + 1$ ). Prove that if  $2n$  of the altitudes are concurrent, then the remaining altitude passes through the point of concurrence.