3585. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let $T_n(x)$ be the Chebyshev polynomial of the first kind defined by the recurrence $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$ for $n \geq 1$ and the initial conditions $T_0(x) = 1$ and $T_1(x) = x$. Find all positive integers n such that

$$T_n(x) \leq (2^{n-2}+1)x^n - 2^{n-2}x^{n-1}, \quad x \in [1, \infty).$$

3586. *Proposed by Shai Covo, Kiryat-Ono, Israel.*

For each positive integer n, a_n is the number of positive divisors of n of the form $4m+1$ minus the number of positive divisors of n of the form $4m+3$ (so $a_4 = 1$, $a_5 = 2$, and $a_6 = 0$). Evaluate the sum $\sum\limits_{n=1}^{\infty} (-1)^{n+1}\frac{a_n}{n}$.

3587★. *Proposed by Ignotus, Colegio Manablanca, Facatativá, Colombia.*

Define the *prime graph* of a set of positive integers as the graph obtained by letting the numbers be the vertices, two of which are joined by an edge if and only if their sum is prime.

(a) Prove that given any tree T on n vertices, there is a set of positive integers whose prime graph is isomorphic to T.

(b) For each positive integer n, determine $t(n)$, the smallest number such that for any tree T on n vertices, there is a set of n positive integers each not greater than $t(n)$ whose prime graph is isomorphic to T.

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3576. *Proposé par Mehmet Şahin, Ankara, Turquie.*

Soit D, E et F trois points à l'intérieur d'un triangle ABC de sorte que $\angle FAB = \angle EAC$, $\angle FBA = \angle DBC$, $\angle DCB = \angle ECA$, $AF = AE$, $BF = BD$ et $CD = CE$. Si R et r sont respectivement les rayons du cercle exinscrit de ABC et de EDF, et s le demi-périmètre de ABC, montrer que l'aire du triangle EDF est $\frac{sr^2}{2R}$.

3577. *Proposé par Mehmet Şahin, Ankara, Turquie.*

Soit H l'orthocentre du triangle acutangle ABC avec A' sur la demi-droite HA et tel que $A'A = BC$. On définit B' et C' de manière analogue. Montrer que

$$\text{Aire}(A'B'C') = 4\text{Aire}(ABC) + \frac{a^2+b^2+c^2}{2}.$$