3569★. *Proposed by Jian Liu, East China Jiaotang University, Nanchang City, China.*

Let the point P lie inside the triangle ABC and let the point Q lie outside the triangle. Let w_1, w_2, w_3 denote the lengths of the angle bisectors of $\angle BPC$, $\angle CPA$, $\angle APB$, respectively. Does the inequality

$$PA \cdot QA + PB \cdot QB + PC \cdot QC \;\geq\; 4(w_1w_2 + w_2w_3 + w_3w_1)$$

hold? [*At* `http://www.emis.de/journals/JIPAM/article1162.html?sid=1162` *the proposer's inequality is proved when Q lies* inside *the triangle.*]

3570. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let r, r_a, r_b, r_c, and R be, respectively, the inradius, the exradii, and the circumradius of triangle ABC with side lengths a, b, c. Prove that

$$\frac{r_a^2}{a^2+r_a^2} + \frac{r_b^2}{b^2+r_b^2} + \frac{r_c^2}{c^2+r_c^2} \;\geq\; \frac{4R+r}{4R-r}\,.$$

3571. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let $n \geq 1$ be an integer. Among all increasing arithmetic progressions x_1, x_2, $\dots$, x_n such that $x_1^2 + x_2^2 + \cdots + x_n^2 = 1$, find the progression with the greatest common difference d.

3572. *Proposed by José Luis Díaz-Barrero, Universitat Politècnica de Catalunya, Barcelona, Spain.*

Let a, b, c be positive real numbers such that $a+b+c=1$. Prove that

$$\left(\sum_{\text{cyclic}} \frac{ab}{c+ab}\right) \;+\; \frac{1}{4}\prod_{\text{cyclic}}\left(\frac{a+\sqrt{ab}}{a+b}\right) \;\geq\; 1\,.$$

3573. *Proposed by A.A. Dzhumadil'daeva, Almaty Republic Physics and Mathematics School, Almaty, Kazakhstan.*

Let $(2n+1)!! = 1\cdot 3\cdots(2n+1)$ be the double factorial, so (for example) $7!! = 105$. Make the convention that $0!! = (-1)!! = 1$. Prove that for any nonnegative integer n,

$$\sum_{\substack{i+j+k=n\\ i,j,k\geq 0}} \binom{n}{i,j,k}(2i-1)!!(2j-1)!!(2k-1)!! \;=\; (2n+1)!!\,.$$