3. One obtains the equation of the pencil of circles generated by a circle and a line by setting equal to zero a linear combination of the circle's formula and the product of the line's formula times that of the line at infinity: $x+y+z$. (Circles are conics that pass through the conjugate imaginary points on the line at infinity; all circles will contain those two points while the pencil generated by a circle and line will consist of all circles through the common pair of points of that circle and line—points which are possibly imaginary or coincident.)

   From the equation of Γ in (1), we conclude that it is a member in the pencil of circles generated by the circumcircle $a^2yz + b^2zx + c^2xy = 0$ and the perpendicular bisector of II' with equation computed in (2).

   Now, any circle through I and I' is orthogonal to the circumcircle.

   Since the isodynamic points are inverse in the circumcircle, the circle through I, I' and one of them must also contain the other. In other words, the circle $\mathcal{C}$ through I and the isodynamic points contains I', and is orthogonal to every circle in the pencil generated by the circumcircle and the perpendicular bisector of II'.

   Since Γ is a member of this pencil, it is orthogonal to circle $\mathcal{C}$.

No other solutions were received.

3556. [2010 : 315, 317] *Proposed by Arkady Alt, San Jose, CA, USA.*

For any acute triangle with side lengths a, b, and c, prove that

$$(a+b+c)\min\{a,b,c\} \ \le\ 2ab+2bc+2ca-a^2-b^2-c^2\,.$$

I. Solution by Edmund Swylan, Riga, Latvia.

Note that $a+b+c = (-a+b+c)+(a-b+c)+(a+b-c)$, where each of the three terms on the right are positive. Now,

$$\begin{aligned} -a^2+ab+ca &= (-a+b+c)a, \\ ab-b^2+bc &= (a-b+c)b, \\ ca+bc-c^2 &= (a+b-c)c. \end{aligned}$$

The result follows, for any triangle, by adding these equations, replacing the final factors on the right by $\min\{a,b,c\}$, and using the first identity above.

II. Solution by Chip Curtis, Missouri Southern State University, Joplin, MO, USA.

Taking $a \le b \le c$, the required inequality may be successively rewritten:

$$\begin{aligned} ab+2bc+ca-2a^2-b^2-c^2 &\ge 0, \\ 2a(b-a)+(c-b)(a+b-c) &\ge 0. \end{aligned}$$

By the triangle inequality, both terms are nonnegative, proving the claim.

III. Solution by Paolo Perfetti, Dipartimento di Matematica, Università degli studi di Tor Vergata Roma, Rome, Italy

We introduce the well-known change of variables $a = x + z$, $b = x + y$, $c = y + z$, or $x = \frac{a+b-c}{2} \geq 0$, $y = \frac{b+c-a}{2} \geq 0$ and $z = \frac{a+c-b}{2} \geq 0$. Taking $a \leq b \leq c$ gives $y \geq x, z$ and the required inequality becomes

$$2(x + y + z)(x + z) \leq 4(xy + yz + xz),$$

which is equivalent to $(xy+zy) \geq x^2+z^2$, which holds because of the inequalities between x, y, z. The result follows.

Also solved by ŠEFKET ARSLANAGIĆ, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; GEORGE APOSTOLOPOULOS, Messolonghi, Greece; ROY BARBARA, Lebanese University, Fanar, Lebanon; MICHEL BATAILLE, Rouen, France; PRITHWIJIT DE, Homi Bhabha Centre for Science Education, Mumbai, India; OLIVER GEUPEL, Brühl, NRW, Germany; JOE HOWARD, Portales, NM, USA; KEE-WAI LAU, Hong Kong, China; SALEM MALIKIĆ, student, Sarajevo College, Sarajevo, Bosnia and Herzegovina; CRISTINEL MORTICI, Valahia University of Târgovişte, Romania; ALBERT STADLER, Herrliberg, Switzerland; PETER Y. WOO, Biola University, La Mirada, CA, USA; TITU ZVONARU, Comǎneşti, Romania; and the proposer.

Most solvers observed that the result holds regardless of whether the triangle is acute. Arslanagić, Geupel and Malikić also noted the case of equality, $a = b = c$, but all appear to have missed two separate (degenerate) cases of equality, namely (i) $c = 2a = 2b$ and (ii) $a = 0$, $b = c$.

3563. [2010 : 316, 318] *Proposed by Mikhail Kochetov and Sergey Sadov, Memorial University of Newfoundland, St. John's, NL.*

A square $n \times n$ array of lamps is controlled by an $n \times n$ switchboard. Flipping a switch in position (i, j) changes the state of all lamps in row i and in column j.

(a) Prove that for even n it is possible to turn off all the lamps no matter what the initial state of the array is. Demonstrate how to do it with the minimum number of switches.

(b) Prove that for odd n it is possible to turn off all the lamps if and only if the initial state of the array has the following property: either the number of ON lamps in every row and every column is odd, or the number of ON lamps in every row and every column is even. If this property holds, provide an algorithm to turn off all the lamps.

Solution by Steffen Weber, student, Martin-Luther-Universität, Halle, Germany.

The order in which switches are flipped does not affect the final state, and flipping a switch twice has no effect. A series of flips, avoiding this redundancy, may be coded as an $n \times n$ $(0, 1)$-matrix in which "1" represents a flip in the corresponding position. It follows that there are at most 2^{n^2} distinct transformations.