3555. *Proposed by Vahagn Aslanyan, Yerevan, Armenia.*

Let a and b be positive integers, $1 < a < b$, such that a does not divide b. Prove that there exists an integer x such that $1 < x \le a$ and both a and b divide $x^{\phi(b)+1} - x$, where ϕ is Euler's totient function.

3556. *Proposed by Arkady Alt, San Jose, CA, USA.*

For any acute triangle with side lengths a, b, and c, prove that

$$(a+b+c)\min\{a,b,c\} \;\le\; 2ab+2bc+2ca-a^2-b^2-c^2\,.$$

3557. *Proposed by Paolo Perfetti, Dipartimento di Matematica, Università degli studi di Tor Vergata Roma, Rome, Italy.*

Let $\{a_k\}_{k=1}^{\infty}$ be a sequence of positive real numbers with $\sum\limits_{k=1}^{\infty} a_k = 1$ and $a_{k+1} \le \dfrac{a_k}{1-a_k}$. Let $S_n^{(p)} = \left(\sum\limits_{k=1}^{n} a_k^p\right)^{1/p}$, and for $p \ge 1$ prove that

$$\lim_{n\to\infty} \sum_{k=n+1}^{n} \frac{k}{n} \left(\prod_{j=1}^{n} \frac{j^{1/p} a_{k+j} a_j}{S_{k+j}^{(p)}}\right)^{1/n} \;=\; 0\,.$$

3558. *Proposed by Johan Gunardi, student, SMPK 4 BPK PENABUR, Jakarta, Indonesia.*

Given two distinct positive integers a and b, prove that there exists a positive integer n such that an and bn have different numbers of digits.

3559★. *Proposed by Thanos Magkos, 3rd High School of Kozani, Kozani, Greece.*

Let ABC be a triangle with side lengths a, b, c, inradius r, circumradius R, and semiperimeter s. Prove that

$$\frac{(b+c)^2}{4bc} \;\le\; \frac{s^2}{3r(4R+r)}\,.$$

3560. *Proposed by Pham Van Thuan, Hanoi University of Science, Hanoi, Vietnam.*

Let x and y be real numbers such that $x^2+y^2=1$. Find the maximum value of

$$f(x,y) \;=\; |x-y| + \left|x^3-y^3\right|\,.$$