3329. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let r be a real number, $0 < r \leq 1$, and let x, y, and z be positive real numbers such that $xyz = r^3$. Prove that

$$\frac{1}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+y^2}} + \frac{1}{\sqrt{1+z^2}} \;\leq\; \frac{3}{\sqrt{1+r^2}}\,.$$

3330. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let n be a natural number, let r be a real number, and let $a_1, a_2, \ldots, a_n$ be positive real numbers satisfying $\prod\limits_{k=1}^{n} a_k = r^n$; prove that

$$\sum_{k=1}^{n} \frac{1}{(1+a_k)^3} \;\geq\; \frac{n}{(1+r)^3}\,,$$

(a) for $n = 2$ if and only if $r \geq \frac{1}{3}$;

(b) for $n = 3$ if $r \geq \frac{1}{\sqrt[3]{4}}$;

(c) for $n = 4$ if $r \geq \frac{1}{\sqrt[3]{4}}$;

(d) for $n \geq 5$ if and only if $r \geq \sqrt[3]{n} - 1$.

3331. *Proposed by José Gibergans-Báguena and José Luis Díaz-Barrero, Universitat Politècnica de Catalunya, Barcelona, Spain.*

Let a, b, and c be the lengths of the sides of triangle ABC, and let R be its circumradius. Prove that

$$\sqrt[3]{a^2b} + \sqrt[3]{b^2c} + \sqrt[3]{c^2a} \;\leq\; 3\sqrt{3}\,R\,.$$

3332. *Proposed by Panos E. Tsaoussoglou, Athens, Greece.*

Let a_1, a_2, a_3, and a_4 be positive real numbers and let λ and μ be positive integers.

(a) Prove that

$$\frac{a_1}{\lambda a_2 + \mu a_3} + \frac{a_2}{\lambda a_3 + \mu a_1} + \frac{a_3}{\lambda a_1 + \mu a_2} \;\geq\; \frac{3}{\lambda + \mu}\,.$$

(b) Prove that

$$\frac{a_1}{\mu a_2 + \mu a_3 + \mu a_4} + \frac{a_2}{\lambda a_3 + \lambda a_4 + \lambda a_1} + \frac{a_3}{\mu a_4 + \lambda a_1 + \mu a_2} + \frac{a_4}{\lambda a_1 + \mu a_2 + \lambda a_3} \;\geq\; \frac{8}{3(\lambda + \mu)}\,.$$