3319. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let m be a natural number, $m \geq 2$, and let r be any real number such that $r \geq 1/m$. If a and b are positive real numbers satisfying $ab = r^2$, prove that

$$\frac{1}{(1+a)^m} + \frac{1}{(1+b)^m} \geq \frac{2}{(1+r)^m}.$$

3320. *Proposed by Michel Bataille, Rouen, France.*

Let $\triangle ABC$ be right-angled at A and let O be the mid-point of BC. Let M be a point in the plane of $\triangle ABC$, and let M', M'', N, N', and N'' denote the orthocentres of $\triangle MAB$, $\triangle MAC$, $\triangle AM'M''$, $\triangle NAB$, and $\triangle NAC$, respectively. If O is the mid-point of $M'M''$, show that O is also the mid-point of $N'N''$.

3321. *Proposed by Michel Bataille, Rouen, France.*

Let the incircle of $\triangle ABC$ have centre I and meet the sides AC and AB at E and F, respectively. For a point M on the line segment EF, show that $\triangle MAB$ and $\triangle MCA$ have the same area if and only if $MI \perp BC$.

3322. *Proposed by Panos E. Tsaoussoglou, Athens, Greece.*

Let a, b, and c be non-negative real numbers such that $a \leq b \leq c$, and let n be a positive integer. Prove that

$$\big(a+(n+1)b\big)\big(b+(n+2)c\big)(c+na) \geq (n+1)(n+2)(n+3)abc.$$

3323. *Proposed by Panos E. Tsaoussoglou, Athens, Greece.*

Let a, b, and c be non-negative real numbers with $a^2+b^2+c^2=1$. Prove that

$$\sum_{\text{cyclic}}(1-2a^2)(b-c)^2 \geq 0.$$

3324. *Proposed by Panos E. Tsaoussoglou, Athens, Greece.*

Let a, b, and c be non-negative real numbers with $a^2+b^2+c^2=1$. Prove that

$$3-5(ab+bc+ca)+6abc(a+b+c) \geq 0.$$

3325. *Proposed by Manuel Benito Muñoz, IES P.M. Sagasta, Logroño, Spain.*

Let $\sigma(n)$ denote the sum of the divisors of the natural number n.

(a) Find a natural number n such that

$$\sigma(n)+500 = \sigma(n+2).$$

(b)★ How many solutions are there to part (a)?