

**3298.** Proposed by Stanley Rabinowitz, MathPro Press, Chelmsford, MA, USA.

Let  $ABC$  be a triangle of area  $\frac{1}{2}$  in which  $a$  is the length of the side opposite vertex  $A$ . Prove that

$$a^2 + \csc A \geq \sqrt{5}.$$

[Ed.: The proposer's only proof of this is by computer. He is hoping that some *CRUX with MAYHEM* reader will find a simpler solution.]

**3299.** Proposed by Victor Oxman, Western Galilee College, Israel.

Given positive real numbers  $a$ ,  $b$ , and  $w_b$ , show that

- (a) if a triangle  $ABC$  exists with  $BC = a$ ,  $CA = b$ , and the length of the interior bisector of angle  $B$  equal to  $w_b$ , then it is unique up to isomorphism;
- (b) for the existence of such a triangle in (a), it is necessary and sufficient that

$$b > \frac{2a|a - w_b|}{2a - w_b} \geq 0;$$

- (c) if  $h_a$  is the length of the altitude to side  $BC$  in such a triangle in (a), we have  $b > |a - w_b| + \frac{1}{2}h_a$ .

**3300.** Proposed by Arkady Alt, San Jose, CA, USA.

Let  $a$ ,  $b$ , and  $c$  be positive real numbers. For any positive integer  $n$  define

$$F_n = \left( \frac{3(a^n + b^n + c^n)}{a + b + c} - \sum_{\text{cyclic}} \frac{b^n + c^n}{b + c} \right).$$

- (a) Prove that  $F_n \geq 0$  for  $n \leq 5$ .
- (b)★ Prove or disprove that  $F_n \geq 0$  for  $n \geq 6$ .

