3182. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let $a = BC$, $b = CA$, and $c = AB$ in $\triangle ABC$. Prove that

$$\frac{bc}{b+c}\sin^2\left(\frac{A}{2}\right) + \frac{ca}{c+a}\sin^2\left(\frac{B}{2}\right) + \frac{ab}{a+b}\sin^2\left(\frac{C}{2}\right) \;\le\; \frac{a+b+c}{8}\,.$$

3183. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let ABC be a triangle with inradius r and circumradius R. If s is the semiperimeter of the triangle, prove that

$$\sqrt{3}\,s \;\le\; r + 4R\,.$$

3184. *Proposed by Fabio Zucca, Politecnico di Milano, Milano, Italy.*

For any real number x, let (x) denote the *fractional part* of x; that is, $(x) = x - \lfloor x \rfloor$, where $\lfloor x \rfloor$ is the greatest integer not exceeding x. Given $n \in \mathbb{Z}$, find all solutions of the equation

$$(x^2) - n(x) \;=\; 0\,.$$

3185. *Proposed by Panos E. Tsaoussoglou, Athens, Greece.*

Let n be an integer, $n \ge 2$, and let a, b, and c be positive real numbers satisfying $a^2 + b^2 + c^2 = 1$. Prove that

$$\frac{a}{1-a^n} + \frac{b}{1-b^n} + \frac{c}{1-c^n} \;\ge\; \frac{(n+1)^{1+\frac{1}{n}}}{n}\,.$$

3186. *Proposed by Vasile Cîrtoaje, University of Ploiesti, Romania.*

Let $f(x)$ be a function on an interval I which is convex for $x \ge a$ for some $a \in I$. Suppose that for all $x_1, x_2, \ldots, x_n \in I$ which satisfy $x_1 + x_2 + \cdots + x_n = na$, the following inequality holds:

$$\frac{f(x_1) + f(x_2) + \cdots + f(x_n)}{n} \;\ge\; f\left(\frac{x_1 + x_2 + \cdots + x_n}{n}\right)\,.$$

Prove that this same inequality holds for all $x_1, x_2, \ldots, x_n \in I$ such that $x_1 + x_2 + \cdots + x_n \ge na$.

3187. *Proposed by Michel Bataille, Rouen, France.*

Let $ABCD$ be a planar quadrilateral which is not a parallelogram. Let C' and D' be the orthogonal projections onto the line AB of the points C and D, respectively. The perpendiculars from C to AD and from D to BC meet at P; the perpendiculars from C' to AD and from D' to BC meet at Q. Show that PQ is perpendicular to the line through the mid-points of AC and BD.