PROBLEMS

Solutions to problems in this issue should arrive no later than **1 August 2007**. *An asterisk* (⋆) *after a number indicates that a problem was proposed without a solution.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English. In the solutions section, the problem will be stated in the language of the primary featured solution.

The editor thanks Jean-Marc Terrier and Martin Goldstein of the University of Montreal for translations of the problems.

We have discovered that some recently posed problems are repeats of earlier problems: problem 3182 [2006 : 463, 464] is the same as problem 3096 [2005 : 544, 547]; problem 3185 [2006 : 463, 465] is the same as problem 2935 [2004 : 174, 176], and problem 3198 [2006 : 516, 518] is the same problem as 3187 [2006 : 463, 465]. Since all three duplications have only appeared within the last two issues, we are replacing them in this issue. Any solutions for the original problems 3182 and 3185 will be ignored, since solutions to those problems have already appeared; any solutions to the original 3198 will be treated as solutions to 3187. Our thanks to Michel Bataille for bringing this to our attention.

3182. Replacement. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let a, b, and c be any positive real numbers, and let p be a real number such that $0 < p < 1$.

(a) Prove that

$$\frac{a}{(b+c)^p}+\frac{b}{(c+a)^p}+\frac{c}{(a+b)^p} \;\ge\; \frac{1}{2^p}\left(a^{1-p}+b^{1-p}+c^{1-p}\right)\,.$$

(b) Prove that, if $p = 1/3$, then

$$\frac{a}{(a+b)^p}+\frac{b}{(b+c)^p}+\frac{c}{(c+a)^p} \;\ge\; \frac{1}{2^p}\left(a^{1-p}+b^{1-p}+c^{1-p}\right)\,.$$

(c)★ Prove or disprove

$$\frac{a}{\sqrt{a+b}}+\frac{b}{\sqrt{b+c}}+\frac{c}{\sqrt{c+a}} \;\ge\; \frac{1}{\sqrt{2}}\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right)\,.$$

3185. Replacement. *Proposed by Shaun White, student, Vincent Massey Secondary School, Windsor, ON.*

Let a_n denote the units digit of $(4n)^{(3n)^{(2n)^n}}$. Find all positive integers n such that $\sum\limits_{i=1}^{n} a_i \ge 4n$.