3165. *Proposed by Mihály Bencze, Brasov, Romania.*

For any positive integer n, prove that there exists a polynomial $P(x)$, of degree at least $8n$, such that

$$\sum_{k=1}^{(2n+1)^2} |P(k)| \;<\; |P(0)| \,.$$

3166. *Proposed by Mihály Bencze and Marian Dinca, Brasov, Romania.*

Let P be an interior point of the triangle ABC. Denote by d_a, d_b, d_c the distances from P to the sides BC, CA, AB, respectively, and denote by D_A, D_B, D_C the distances from P to the vertices A, B, C, respectively. Further let P_A, P_B, and P_C denote the measures of $\angle BPC$, $\angle CPA$, and $\angle APB$, respectively.

Prove that

$$\begin{aligned} d_ad_b\sin\left(\tfrac{1}{2}(P_A+P_B)\right) + d_bd_c\sin\left(\tfrac{1}{2}(P_B+P_C)\right) + d_cd_a\sin\left(\tfrac{1}{2}(P_C+P_A)\right) \\ \leq\; \tfrac{1}{4}(D_BD_C\sin P_A + D_CD_A\sin P_B + D_AD_B\sin P_C)\,. \end{aligned}$$

3167. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let ABC be a non-obtuse triangle with circumradius R. If a, b, c are the lengths of the sides opposite angles A, B, C, respectively, prove that

$$a\cos^3 A + b\cos^3 B + c\cos^3 C \;\leq\; \frac{abc}{4R^2}\,.$$

3168. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let x_1, x_2, $\dots$, x_n be positive real numbers satisfying $\prod\limits_{i=1}^{n} x_i = 1$. Prove that

$$\sum_{i=1}^{n} x_i^n(1+x_i) \;\geq\; \frac{n}{2^{n-1}}\prod_{i=1}^{n}(1+x_i)\,.$$

3169. *Proposed by Vesselin Dimitrov, National Highschool of Mathematics and Science, Sofia, Bulgaria.*

Let A be a finite set of real numbers such that each $a \in A$ is uniquely expressible as $a = b + c$, where $b, c \in A$ and $b \leq c$.

(a) Prove that there exist distinct elements a_1, a_2, $\dots$, $a_k \in A$ such that $a_1 + a_2 + \cdots + a_k = 0$.

(b)★ Does this necessarily hold if it is no longer assumed that each representation $a = b + c$ is unique?