3116. *Proposed by Arkady Alt, San Jose, CA, USA.*

For arbitrary real numbers a, b, c, prove that

$$\sum_{\text{cyclic}} a(b+c-a)^3 \;\leq\; 4abc(a+b+c)\,.$$

3117. *Proposed by Li Zhou, Polk Community College, Winter Haven, FL, USA.*

Let a, b, c be the lengths of the sides and s the semi-perimeter of $\triangle ABC$. Prove that

$$\sum_{\text{cyclic}} (a+b)\sqrt{ab(s-a)(s-b)} \;\leq\; 3abc\,.$$

3118. *Proposed by D.J. Smeenk, Zaltbommel, the Netherlands.*

Let BE and CF be altitudes of the acute-angled triangle ABC with E on AC and F on AB. Let BK and CL be the interior angle bisectors of $\angle ABC$ and $\angle ACB$, respectively, with K on AC and L on AB. Let I denote the incentre of $\triangle ABC$, and let O denote its circumcentre. Prove that E, F, and I are collinear if and only if K, L, and O are collinear.

3119. *Proposed by Michel Bataille, Rouen, France.*

Let r and s denote the inradius and semi-perimeter, respectively, of triangle ABC. Show that

$$3\sqrt{3}\sqrt{\frac{r}{s}} \;\leq\; \sqrt{\tan\left(\tfrac{1}{2}A\right)} + \sqrt{\tan\left(\tfrac{1}{2}B\right)} + \sqrt{\tan\left(\tfrac{1}{2}C\right)} \;\leq\; \sqrt{\frac{s}{r}}\,.$$

3120. *Proposed by Michel Bataille, Rouen, France.*

Let ABC be an isosceles triangle with $AB = BC$, and let F be the mid-point of AC. Let $\alpha = \angle BAX$, where X is a variable point on the ray BF. As long as $\alpha \neq \pi/2$, the reflections of the line BF in BA and XA intersect. Let that point of intersection be denoted by M.

Find $\displaystyle\lim_{\alpha\to\pi/2} |\cos\alpha|\cdot CM$.

3121. *Proposed by José Luis Díaz-Barrero, Universitat Politècnica de Catalunya, Barcelona, Spain.*

Let n and r be positive integers. Show that

$$\left(\frac{1}{2^n}\sum_{k=1}^{n}\frac{1}{k}\binom{n-1}{k-1}\left[1-\frac{1}{2^{nr}}\binom{n}{k}^r\right]\right)^r \;\leq\; \frac{r^r}{(r+1)^{r+1}}\,.$$