with equality if and only if $x = 1/n$.

Finally, apply (2) to $x_0, x_1, \ldots, x_n$ to obtain

$$\sum_{k=0}^{n} \frac{1}{n^2 x_k + 1} \geq \sum_{k=0}^{n} \frac{1}{x_k + 1} - (n+1)\frac{n^2-1}{(n+1)^2} = n - \frac{n^2-1}{n+1} = 1,$$

with equality if and only if $x_0 = x_1 = \cdots = x_n = 1/n$.

Also solved by ARKADY ALT, San Jose, CA, USA; ROY BARBARA, University of Beirut, Beirut, Lebanon; MICHEL BATAILLE, Rouen, France; MANUEL BENITO, ÓSCAR CIAURRI, and EMILIO FERNÁNDEZ, Logroño, Spain; JIM BLACK, student, Missouri State University, Springfield, MO, USA; CHIP CURTIS, Missouri Southern State University, Joplin, MO, USA; APOSTOLIS K. DEMIS, Varvakeio High School, Athens, Greece; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; JOE HOWARD, Portales, NM, USA; WALTHER JANOUS, Ursulinengymnasium, Innsbruck, Austria; KEE-WAI LAU, Hong Kong, China; DRAGOLJUB MILOŠEVIĆ and G. MILANOVAC, Serbia; VEDULA N. MURTY, Dover, PA, USA; CAO MINH QUANG, Nguyen Binh Khiem specialized high school, Vinh Long, Vietnam; JOEL SCHLOSBERG, Bayside, NY, USA; PETER Y. WOO, Biola University, La Mirada, CA, USA; LI ZHOU, Polk Community College, Winter Haven, FL, USA; TITU ZVONARU, Comănești, Romania; and the proposer.

About half of the solvers used calculus or convexity and Jensen's Inequality. Zhou showed that the result is actually true for a, b, $c \in (-\infty, -1) \cup (-1/4, \infty)$. Several other generalizations were obtained. Benito, Ciaurri, and Fernández proved that if $n \geq 3$ and $a_1, \ldots, a_n$ are positive real numbers such that $\sum\limits_{i=1}^{n} \frac{1}{a_i+1} = 2$, then $\sum\limits_{i=1}^{n} \frac{1}{k^2 a_i + 1} \geq 1$, for $k = \frac{n+1}{n-1}$. Their proof is a straightforward generalization of Solution I above. Janous proved that if $n \geq 2$ and $x_1, x_2, \ldots, x_n$ are positive real numbers such that $\sum\limits_{i=1}^{n} \frac{1}{x_i+1} = a$, where $a < n$ is a constant, then $\sum\limits_{i=1}^{n} \frac{1}{bx_i+1} \geq \frac{an}{b(n-a)+a}$ for all constants $b > 1$. The special case when $n = 3$, $a = 2$, and $b = 4$ is the proposed inequality. Quang established the similar result that if $\sum\limits_{i=1}^{n} \frac{1}{x_i+1} \geq 1$, then $\sum\limits_{i=1}^{n} \frac{1}{4x_i+1} \geq \frac{n}{4n-3}$.

3115. [2006 : 107, 109] *Proposed by Arkady Alt, San Jose, CA, USA.*

Let a, b, c, be the lengths of the sides opposite the vertices A, B, C, respectively, in triangle ABC. Prove that

$$\frac{\cos^3 A}{a} + \frac{\cos^3 B}{b} + \frac{\cos^3 C}{c} < \frac{a^2+b^2+c^2}{2abc}.$$

Essentially the same solution by Charles R. Diminnie, Angelo State University, San Angelo, TX, USA; and Li Zhou, Polk Community College, Winter Haven, FL, USA.

Let R be the circumradius of $\triangle ABC$. By the Law of Sines, we have

$$\begin{aligned}
&\sum_{\text{cyclic}} (b^2+c^2-a^2)\sin^2 A \\
&= \sum_{\text{cyclic}} \frac{a^2(b^2+c^2-a^2)}{4R^2} = \frac{2(a^2b^2+b^2c^2+c^2a^2)-(a^4+b^4+c^4)}{4R^2} \\
&= \frac{(a+b+c)(a+b-c)(b+c-a)(c+a-b)}{4R^2} > 0.
\end{aligned}$$

Hence,

$$\sum_{\text{cyclic}} (b^2+c^2-a^2)\cos^2 A \;=\; \sum_{\text{cyclic}} (b^2+c^2-a^2)(1-\sin^2 A)$$
$$<\; \sum_{\text{cyclic}} (b^2+c^2-a^2) \;=\; a^2+b^2+c^2\,,$$

which is equivalent to $\sum\limits_{\text{cyclic}} \left(2bc\cos^3 A\right) < a^2+b^2+c^2$. Dividing both sides by $2abc$, the result follows immediately.

Also solved by MOHAMMED AASSILA, Strasbourg, France; ŠEFKET ARSLANAGIĆ, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; ROY BARBARA, University of Beirut, Beirut, Lebanon; MICHEL BATAILLE, Rouen, France; MANUEL BENITO, ÓSCAR CIAURRI, and EMILIO FERNÁNDEZ, Logroño, Spain; APOSTOLIS K. DEMIS, Varvakeio High School, Athens, Greece; WALTHER JANOUS, Ursulinengymnasium, Innsbruck, Austria; VEDULA N. MURTY, Dover, PA, USA; ALEX REMOROV, student, William Lyon Mackenzie Collegiate Institute, Toronto, ON; JUAN-BOSCO ROMERO MÁRQUEZ, Universidad de Valladolid, Valladolid, Spain; JOEL SCHLOSBERG, Bayside, NY, USA; PETER Y. WOO, Biola University, La Mirada, CA, USA; TITU ZVONARU, Comănești, Romania; and the proposer. There were also two incorrect solutions.

Both Janous and Zvonaru showed that the given inequality is equivalent to

$$\sum_{\text{cyclic}} a^2(b^2+c^2-a^2)^3 \;<\; 4a^2b^2c^2(a^2+b^2+c^2)\,,$$

and remarked that this is a special case of Crux problem #3116 (by the same proposer). Zvonaru also pointed out that if $\triangle ABC$ is an acute triangle, then the following is a very simple proof of the given inequality:

$$\sum_{\text{cyclic}} \frac{\cos^3 A}{a} \;<\; \sum_{\text{cyclic}} \frac{\cos A}{a} \;=\; \sum_{\text{cyclic}} \frac{b^2+c^2-a^2}{2abc} \;=\; \frac{a^2+b^2+c^2}{2abc}\,.$$

3116. [2006 : 107, 110] *Proposed by Arkady Alt, San Jose, CA, USA.*

For arbitrary real numbers a, b, c, prove that

$$\sum_{\text{cyclic}} a(b+c-a)^3 \;\le\; 4abc(a+b+c)\,.$$

Essentially the same solution by Šefket Arslanagić, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; Michel Bataille, Rouen, France; and Joel Schlosberg, Bayside, NY, USA.

$$4abc(a+b+c)-\sum_{\text{cyclic}} a(b+c-a)^3 \;=\; (a^2+b^2+c^2-2ab-2ac-2bc)^2 \;\ge\; 0\,.$$

The equality holds if and only if $a=b$ and $c=0$, $b=c$ and $a=0$, or $c=a$ and $b=0$.

Also solved by ROY BARBARA, University of Beirut, Beirut, Lebanon; CHIP CURTIS, Missouri Southern State University, Joplin, MO, USA; VEDULA N. MURTY, Dover, PA, USA; PETER Y. WOO, Biola University, La Mirada, CA, USA; LI ZHOU, Polk Community College, Winter Haven, FL, USA; TITU ZVONARU, Comănești, Romania; and the proposer.