3096. [2005 : 544, 547] *Proposed by Arkady Alt, San Jose, CA, USA.*

Let ABC be a triangle with sides a, b, c opposite the angles A, B, C, respectively. Prove that

$$\sum_{\text{cyclic}} \frac{bc}{b+c}\sin^2\left(\frac{A}{2}\right) \;\leq\; \frac{a+b+c}{8}\,.$$

Similar solutions by Vedula N. Murty, Dover, PA, USA; and Li Zhou, Polk Community College, Winter Haven, FL, USA.

Since $\frac{bc}{b+c} \leq \frac{b+c}{4}$, $\sin^2\frac{A}{2} = \frac{1-\cos A}{2}$, and $a = b\cos C + c\cos B$, we obtain

$$\begin{aligned} 8\sum_{\text{cyclic}} \frac{bc}{b+c}\sin^2\frac{A}{2} - \sum_{\text{cyclic}} a \;&\leq\; \sum_{\text{cyclic}} [(b+c)(1-\cos A)] - \sum_{\text{cyclic}} a \\ &=\; \sum_{\text{cyclic}} a - \sum_{\text{cyclic}} (b\cos C + c\cos B) \;=\; 0\,, \end{aligned}$$

which yields the desired inequality.

Also solved by ŠEFKET ARSLANAGIĆ, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; MICHEL BATAILLE, Rouen, France; CHIP CURTIS, Missouri Southern State University, Joplin, MO, USA; JOHN G. HEUVER, Grande Prairie, AB; WALTHER JANOUS, Ursulinengymnasium, Innsbruck, Austria; PETER Y. WOO, Biola University, La Mirada, CA, USA; BIN ZHAO, YunYuan HuaZhong University of Technology and Science, Wuhan, Hubei, China; and the proposer.

3097. [2005 : 545, 547] *Proposed by Mihály Bencze, Brasov, Romania.*

Let a and b be two positive real numbers such that $a < b$. Define $A(a,b) = \frac{a+b}{2}$ and $L(a,b) = \frac{b-a}{\ln b - \ln a}$. Prove that

$$L(a,b) \;<\; L\left(\frac{a+b}{2}, \sqrt{ab}\right) \;<\; \left(A(\sqrt{a},\sqrt{b})\right)^2 \;<\; A(a,b)\,.$$

Solution by Li Zhou, Polk Community College, Winter Haven, FL, USA.

The inequality $\left(A(\sqrt{a},\sqrt{b})\right)^2 < A(a,b)$ is simply the Power–Mean Inequality. Applying the Hadamard's Inequality to the convex function $f(x) = 1/x$, we get

$$\begin{aligned} \frac{1}{L\left(\frac{1}{2}(a+b), \sqrt{ab}\right)} \;&=\; \frac{1}{\frac{1}{2}(a+b) - \sqrt{ab}} \int_{\sqrt{ab}}^{\frac{1}{2}(a+b)} f(x)\,dx \\ &>\; f\left(\frac{\frac{1}{2}(a+b)+\sqrt{ab}}{2}\right) \;=\; \frac{1}{\left(A(\sqrt{a},\sqrt{b})\right)^2}\,. \end{aligned}$$