if $S \geq rn$. Thus, $\ln P \;\leq\; n\ln\left(1+(S/n)^2\right)$ for $S \geq rn$, and therefore the inequality in (b) holds for $S \geq rn$. The bound on S here is an improvement of the given bound, since $2\sqrt{2}(n-1)/n \geq 3\sqrt{2}/2 > 2 > r$ for $n \geq 4$.

(c) By the AM–GM Inequality, $x_1x_2 \;\leq\; \left(\frac{x_1+x_2}{2}\right)^2 \;\leq\; \left(\frac{S}{2}\right)^2 \;\leq\; 2$. Hence,

$$(1+x_1^2)(1+x_2^2) - \left[1+(x_1+x_2)^2\right] \;=\; x_1x_2(x_1x_2-2) \;\leq\; 0\,.$$

An easy induction completes the proof.

Reference:

[1] G.-Z. Chang and T.W. Sederberg, "Over and Over Again", MAA, 1997, 29–31.

Also solved by WALTHER JANOUS, Ursulinengymnasium, Innsbruck, Austria (*part* (c) *only*)*; and the proposer.*

3095. [2005 : 544, 547] *Proposed by Arkady Alt, San Jose, CA, USA.*

Let a, b, c, p, and q be natural numbers. Using $\lfloor x\rfloor$ to denote the integer part of x, prove that

$$\min\left\{a,\left\lfloor\frac{c+pb}{q}\right\rfloor\right\} \;\leq\; \left\lfloor\frac{c+p(a+b)}{p+q}\right\rfloor\,.$$

Solution by Joel Schlosberg, Bayside, NY, USA.

We have

$$\begin{aligned}
\frac{c+p(a+b)}{p+q} &= \frac{pa}{p+q}+\frac{c+pb}{p+q} \;=\; \frac{p}{p+q}a+\frac{q}{p+q}\frac{c+pb}{q} \\
&\geq \frac{p}{p+q}a+\frac{q}{p+q}\left\lfloor\frac{c+pb}{q}\right\rfloor \\
&\geq \frac{p}{p+q}\min\left\{a,\left\lfloor\frac{c+pb}{q}\right\rfloor\right\}+\frac{q}{p+q}\min\left\{a,\left\lfloor\frac{c+pb}{q}\right\rfloor\right\} \\
&= \min\left\{a,\left\lfloor\frac{c+pb}{q}\right\rfloor\right\}\,.
\end{aligned}$$

Since $\min\left\{a,\left\lfloor\frac{c+pb}{q}\right\rfloor\right\}$ is an integer,

$$\left\lfloor\frac{c+p(a+b)}{p+q}\right\rfloor \;\geq \min\left\{a,\left\lfloor\frac{c+pb}{q}\right\rfloor\right\}\,.$$

Also solved by ŠEFKET ARSLANAGIĆ, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; MICHEL BATAILLE, Rouen, France; CHIP CURTIS, Missouri Southern State University, Joplin, MO, USA; WALTHER JANOUS, Ursulinengymnasium, Innsbruck, Austria; LI ZHOU, Polk Community College, Winter Haven, FL, USA; and the proposer.