3093. *Proposed by Mihály Bencze, Brasov, Romania.*

Let p_k be the k^{th} prime. Show that the following series diverges:

$$\sum_{n=1}^{\infty} \frac{1}{n(p_{n+1} - p_n)} .$$

3094. *Proposed by Vasile Cîrtoaje, University of Ploiesti, Romania.*

Let $x_1, x_2, \ldots, x_n$ be non-negative real numbers, where $n \geq 3$. Let $S = \sum\limits_{k=1}^{n} x_k$ and $P = \prod\limits_{k=1}^{n} (1 + x_k^2)$. Prove that

(a) $P \leq \max\limits_{1 \leq k \leq n} \left\{ \left(1 + \dfrac{S^2}{k^2}\right)^k \right\}$;

(b) $P \leq \left(1 + \dfrac{S^2}{n^2}\right)^n$ if $S > 2\sqrt{2}(n-1)$;

(c) $P \leq 1 + S^2$ if $S \leq 2\sqrt{2}$.

3095. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let a, b, c, p, and q be natural numbers. Using $\lfloor x \rfloor$ to denote the integer part of x, prove that

$$\min\left\{ a, \left\lfloor \frac{c + pb}{q} \right\rfloor \right\} \leq \left\lfloor \frac{c + p(a+b)}{p+q} \right\rfloor .$$

3096. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let ABC be a triangle with sides a, b, c opposite the angles A, B, C, respectively. Prove that

$$\sum_{\text{cyclic}} \frac{bc}{b+c} \sin^2\left(\frac{A}{2}\right) \leq \frac{a+b+c}{8} .$$

3097. *Proposed by Mihály Bencze, Brasov, Romania.*

Let a and b be two positive real numbers such that $a < b$. Define $A(a,b) = \dfrac{a+b}{2}$ and $L(a,b) = \dfrac{b-a}{\ln b - \ln a}$. Prove that

$$L(a,b) < L\left(\frac{a+b}{2}, \sqrt{ab}\right) < \left(A(\sqrt{a}, \sqrt{b})\right)^2 < A(a,b) .$$