3089. [2005 : 543, 546] *Proposed by Christopher J. Bradley, Bristol, UK.*

Let ABC be a triangle and P a point in the plane of this triangle, not lying on any of the three lines determined by its sides. Let AD, BE, and CF be the Cevians through the point P. The lines through E and F parallel to AD meet the line BC at L and L', respectively. Points M, M', N, and N' are similarly defined. Prove that L, L', M, M', N, N' all lie on a conic.

Solution by Li Zhou, Polk Community College, Winter Haven, FL, USA.

Using

$$\frac{BL'}{BD} = \frac{BF}{BA} \quad \text{and} \quad \frac{BD}{BC} = \frac{BN}{BF},$$

we see that

$$\frac{BL'}{BC} = \frac{BL'}{BD} \cdot \frac{BD}{BC} = \frac{BF}{BA} \cdot \frac{BN}{BF} = \frac{BN}{BA}.$$

It follows that $NL' \parallel AC$. Likewise, $LM' \parallel BA$ and $MN' \parallel CB$. Therefore, opposite sides of the hexagon $L'LM'MN'N$ are parallel and thus, its vertices lie on a conic by the converse of Pascal's Theorem.

Also solved by MICHEL BATAILLE, Rouen, France; JOEL SCHLOSBERG, Bayside, NY, USA; PETER Y. WOO, Biola University, La Mirada, CA, USA; and the proposer. There was one incorrect submission.

3090. [2005 : 543, 546] *Proposed by Arkady Alt, San Jose, CA, USA.*

Find all non-negative real solutions (x, y, z) to the following system of inequalities:

$$\begin{aligned} 2x(3-4y) &\geq z^2+1, \\ 2y(3-4z) &\geq x^2+1, \\ 2z(3-4x) &\geq y^2+1. \end{aligned}$$

Solution by Li Zhou, Polk Community College, Winter Haven, FL, USA.

Without loss of generality, we may assume that $x \leq \min\{y, z\}$. From the first equation, we have

$$(3x-1)^2 + 8x(y-x) + (z^2-x^2) = z^2+1-2x(3-4y) \leq 0.$$

Each term of the sum on the left is non-negative; hence, $x = y = z = \frac{1}{3}$.

Also solved by ŠEFKET ARSLANAGIĆ, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; MICHEL BATAILLE, Rouen, France; CHIP CURTIS, Missouri Southern State University, Joplin, MO, USA; WALTHER JANOUS, Ursulinengymnasium, Innsbruck, Austria; JOEL SCHLOSBERG, Bayside, NY, USA; BIN ZHAO, YunYuan HuaZhong University of Technology and Science, Wuhan, Hubei, China; and the proposer.