PROBLEMS

Solutions to problems in this issue should arrive no later than **1 May 2006.** *An asterisk* ($\star$) *after a number indicates that a problem was proposed without a solution.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English. In the solutions section, the problem will be stated in the language of the primary featured solution.

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3076. *Proposed by Vedula N. Murty, Dover, PA, USA.*

If x, y, z are non-negative real numbers and a, b, c are arbitrary real numbers, prove that

$$\big(a(y+z)+b(z+x)+c(x+y)\big)^2 \;\geq\; 4(xy+yz+zx)(ab+bc+ca)\,.$$

(Note: If we impose the conditions that $x+y+z=1$ and that a, b, c are positive, then the above is equivalent to

$$ax+by+cz+2\sqrt{(xy+yz+zx)(ab+bc+ca)} \;\leq\; a+b+c\,,$$

which is problem #8 of the 2001 Ukrainian Mathematical Olympiad, given in the December 2003 issue of ***CRUX with MAYHEM*** [2003 : 498]. The solution of the Ukrainian problem appears on page 443.)

3077. *Proposed by Arkady Alt, San Jose, CA, USA.*

In $\triangle ABC$, we denote the sides BC, CA, AB as usual by a, b, c, respectively. Let h_a, h_b, h_c be the lengths of the altitudes to the sides a, b, c, respectively. Let d_a, d_b, d_c be the signed distances from the circumcentre of $\triangle ABC$ to the sides a, b, c, respectively. (The distance d_a, for example, is positive if and only if the circumcentre and vertex A lie on the same side of the line BC.)

Prove that

$$\frac{h_a+h_b+h_c}{3} \;\leq\; d_a+d_b+d_c\,.$$

3078. *Proposed by D.J. Smeenk, Zaltbommel, the Netherlands.*

Let ABC be a triangle with $a>b$. Let D be the foot of the altitude from A to the line BC, let E be the mid-point of AC, and let CF be an external bisector of $\angle BCA$ with F on the line AB. Suppose that D, E, F are collinear.

(a) Determine the range of $\angle BCA$.

(b) Show that $c>b$.

(c) If $c^2=ab$, determine the measures of the angles of $\triangle ABC$, and show that $\sin B=\cos^2 B$.